On the identities of modulo-$p$ partitions

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Some identities between partitions and compositions were obtained in the literature. As a natural extension, we introduce and study modulo-$p$ partitions, where $p$ is a positive integer. Moreover, several recurrence relations and some sufficient conditions for the existence of modulo-$p$ partitions are given, respectively. In addition, we obtain some identities of modulo-$p$ partitions. In the end, using the properties of a binary tree, we provide a method to determine modulo-$p$ partitions.

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1. Introduction

Lots of interesting partition identities occurred since the first identity was given by Euler [1]. On the other hand, research on identities involving partitions and ordered partitions (namely, compositions [2]) are few and have only occurred in recent years [3–5]. The first such effort was made by Agarwal [3] in 2003, and recently, Guo [4] gives some other identities using Agarwal’s method.

Inspired by the definitions of “odd–even” partitions, “even” partitions and their partition identities in [3,4], we consider a more general problem of the identities between partitions and compositions in this paper.

Throughout this paper, let $s$, $n$, $p$, $m$ be positive integers, and $b$, $q$, $t$, $c$, $r$, $d$ be nonnegative integers such that $s = tp + r$, $n = cp + d$, $0 \leq b, q, r, d \leq p - 1$. Let $x^y$ denote the abbreviation of $x + x + \cdots + x$. We first introduce the (p, b, q)-partition, the F-(p, b, q)-partition and the (p, b)-composition as follows. For convenience, they are all called modulo-p partitions.

**Definition 1.1** ([6]). A two-rowed array of nonnegative integers \( \left( \begin{array}{c} a_1 \\ b_1 \\ a_2 \\ b_2 \\ \vdots \\ a_k \\ b_k \end{array} \right) \) is called a Frobenius partition of $n$, where $a_1 > a_2 > \cdots > a_k \geq 0$, $b_1 > b_2 > \cdots > b_k \geq 0 (k \in \mathbb{Z}_+)$, and $n = k + \sum_{i=1}^{k} a_i + \sum_{i=1}^{k} b_i$.

**Remark 1.** Note that each partition can be represented by a Frobenius notation. For instance, the Frobenius notation of $22 = 8 + 7 + 3 + 3 + 1$ (see the following figure) is \( \left( \begin{array}{c} 7 \\ 4 \\ 5 \\ 2 \\ 0 \end{array} \right) \).

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