Tensor analysis of the instantaneous power in electrical networks

A.J. Ustariz\textsuperscript{a, *}, E.A. Cano\textsuperscript{a}, H.E. Tacca\textsuperscript{b}

\textsuperscript{a} Department of Electrical, Electronic and Computer Engineering, Universidad Nacional de Colombia, Campus La Nubia. A.A. 127, Manizales, Colombia
\textsuperscript{b} Department of Electronic Engineering, Universidad de Buenos Aires, Buenos Aires, Argentina

\section{Introduction}

Power components such as active power, reactive power and apparent power provide the necessary information to design, evaluate, monitor and compensate electrical systems. These powers are thus used to define system characteristics as power factor, installed power capacity, or demanded power.

Nowadays, the components of power are well defined in linear single-phase systems or balanced three-phase linear systems under sinusoidal conditions. However, the increased usage of adjustable speed drives, arc furnaces and single-phase loads like computers and compact fluorescent lamps (CFLs), which are nonlinear loads, distort both the current and voltage waveforms \cite{1}. In this case, the definition of these components is controversial allowing so many approaches, that while some authors restrict their analysis to the frequency domain or time, others formulate broader and more general theories by using the time-frequency domain \cite{2–4}. Fig. 1 shows the theories that led to this division.

In addition, since the beginning of the last century many theories have been proposed to solve the problem, based on some of the approaches already mentioned. These theories, besides choosing a specific approach (frequency, time or time-frequency), have used one of the following trends: (a) theories based on a periodic concept \cite{5–9}, which were developed for single-phase systems and have been implemented mainly for measuring the harmonic content and (b) theories based on an instantaneous concept and on a vector approach \cite{10–16}, which were developed for three-phase systems having as a main objective the design of control algorithms useful for active filters.

This subject is still debated, and there is not yet a generalized power theory that can be assumed as a common base for physical interpretation of power phenomena, power quality assessment and non-active power compensation in multiphase power systems. Therefore, departing from the theories based on an instantaneous concept and on a vector approach, an alternative of instantaneous power calculation and physical interpretation is proposed using the concept of tensor product as a mathematical tool. The “instantaneous power tensor” is a unique compact expression defined using this concept which involves the two components of instantaneous power (active and reactive), in order to geometrically interpret the behavior of electrical phenomena, analogous to deformation formulations in the mechanics of solids.

Also, the mathematical tool here proposed may be easily implemented in the time domain, offering an option for the power quality assessment and non-active power compensation in multiphase power systems.

This paper is organized as follows. Section 2 describes the definition of instantaneous power using a vector analysis. Section 3 introduces the tensor analysis of the instantaneous power. In Section 4 practical applications the tensor analysis are presented. Finally, in Section 5 the most important conclusions are presented.