An improved numerical method for computation of stress intensity factors along 3D curved non-planar cracks in FGMs

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ARTICLE INFO

Article history:
Received 29 May 2010
Received in revised form 11 August 2010
Available online 25 September 2010

Keywords:
FGMs
Arbitrary curved non-planar crack
Stress intensity factors
Interaction integral

Abstract

A simplified strategy based on the interaction energy integral is implemented in the finite element framework to evaluate mixed mode Stress Intensity Factors (SIFs) in 3D non-planar cracks. The proposed approach does not require any a priori information about crack front and crack surface curvatures, therefore different arbitrary non-planar cracks can be easily investigated. In particular, both conical and lens-shaped cracks in homogeneous materials are considered as case studies in order to demonstrate the accuracy of the present approach. Finally, the computational strategy is extended to Functionally Graded Materials (FGMs) and the effect of graded material properties (Young’s modulus and Poisson’s ratio) on the SIFs is studied in detail.

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1. Introduction

The impact of a small hard object on the surface of brittle materials usually leads to the formation of ring cracks and to the occurrence of a conical fracture proceeding downward from the damaged area (Mencik, 1996). Conical cracks also arise in response to indentation loading (Gogotsi, 2009). Due to complexity of non-planar cracks, the analytical solutions for fracture parameters (e.g. SIFs) are limited to some special cases, and therefore additional studies for cracks of general curved shapes are needed. Sládek and Sládek (1983) derived boundary integral solutions to crack problems with cylindrical and spherical crack surfaces. Firth and Keat (1996) obtained some solutions to non-planar crack problems using the surface integral method. As an alternative, Chang and Wu (2007) computed mixed mode SIFs for 3D non-planar cracks by modifying the concept of the Jk and Gmi integrals.

On the other hand, the interaction energy integral allows to make accurate and robust estimates of stress intensity factors. It is derived from the J-integral by considering a composition of two admissible states (Yau et al., 1980). In particular, converting the contour interaction integral to a finite domain surrounding the crack front, singular elements are removed from numerical computation (Nikishkov and Atluri, 1987). As a consequence, this approach does not require to accurately capture singular fields in the vicinity of crack tip, moreover, it can be easily introduced in the finite element context. Gosz and Moran (2002) adopted the interaction integral method to study 3D non-planar cracks in homogeneous materials. In order to define auxiliary fields at integration points for curved non-planar cracks, they employed a curvilinear coordinate system located at integration points. Recently Shaghaghi et al. (submitted for publication) successfully extended the procedure proposed by Gosz and Moran (2002) to 3D non-planar cracks in graded solids. Note that this topic was fairly unexplored because earlier works focused on 3D planar cracks only (Yu et al., 2010; Walters et al., 2004, 2006; Yildirim et al., 2005; Ayhan, 2009, 2007). However, it was observed in Gosz and Moran (2002) that by imposing auxiliary fields in curvilinear coordinate system the auxiliary strain field is not symmetric gradient of auxiliary displacement field and auxiliary stress field is not in equilibrium. Hence special care should be devoted to compensate the influence of crack curvature. For this reason, analytical equations for crack front and crack surface are required to find gradient of auxiliary displacement field, divergence of auxiliary stress field and location of integration points with respect to crack front. Although the previous method has been successfully applied to determine SIFs in homogeneous and FGM materials, in the finite element context it would be more convenient to develop a computational procedure which requires reduced a priori information regarding crack front and crack surface. In such a way any 3D non-planar cracks could be analyzed within the same numerical framework.

Therefore, the aim of the present work is to develop an improved technique, based on the interaction integral in domain form, for computation of mixed mode SIFs along 3D curved non-planar cracks. In addition, the proposed procedure will be also extended to graded materials. To demonstrate the