Nonlinear analysis of elastic space cable-supported membranes

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1. Introduction

Structural membranes play an important role in the engineering field in our days. The lightness of the structure, the ability to cover very large spans and the prefabrication facility are some of their constructional advantages. They are separated into two major categories, the air-supported membranes and the prestressed membranes. The air-supported membrane derives its structural integrity from the use of internal pressurized air to inflate the structural fabric envelope, so that air is the main support of the structure. However, any prestress can be applied to the membrane of the second category by stretching it from its support of the structure. In first instance the cables are assumed undeformed (fixed boundary) and the membrane problem is solved under self-weight and (iii) in-service loading. There are various techniques for the determination of the initial shape of the membrane (e.g. [2–5]). In this investigation the initial form of the membrane is determined by solving the minimal surface problem using the method presented in Ref. [6].

In this paper an iterative solution scheme to the coupled problem of elastic flat or space membranes supported by elastic flexible cables. Both membrane and cable undergo large deflections. Starting from the minimal surface the membrane is prestressed by imposed boundary displacements under the self-weight. Then an iterative procedure is employed, which consists in solving the membrane and the cable large deflection problems separately in each iteration step and checking the continuity of displacements and forces between membrane and cable. The procedure is repeated until convergence is achieved. Both membrane and cable problems are solved using the analog equation method (AEM). The displacements as well as the stress resultants are evaluated at any point of the membrane and the cable from the integral representations of the solution of the analog equations, which are used as mathematical formulae. Example problems are presented, for both flat and space membranes, which illustrate the method and demonstrate its efficiency and accuracy.

In this paper a solution method is presented for the coupled problem of elastic flat or space membranes supported by elastic flexible cables. Both membrane and cable undergo large deflections. However, any prestress can be applied to the membrane of the membrane boundary condition by stretching it from its edges (imposed boundary displacements) or by prestressed cables which support it (cable-supported). The analysis of prestressed membranes involves three steps [1] (i) form-finding, (ii) prestress under self-weight and (iii) in-service loading. There are various techniques for the determination of the initial shape of the membrane (e.g. [2–5]). In this investigation the initial form of the membrane is determined by solving the minimal surface problem using the method presented in Ref. [6].

The deformed membrane and the membrane problem is solved again with fixed boundary taking into account the prestress forces from the previous step. The resulting reactions on the membrane boundary are applied with reversed sign on the cable as external loading and the cable problem is solved. Then the computed displacements of the cable are used as imposed boundary displacements for the membrane and the membrane problem is solved again under the in-service loading. The procedure is repeated until the displacement continuity conditions between cable and membrane are satisfied.

Both membranes and cables exhibit nonlinear behavior due to near zero flexural stiffness which renders them susceptible to large deformations. That is, such structures adapt their shape undergoing large deflections, in order to provide transverse components of the stress resultants to equilibrate the loads. In the present analysis geometric nonlinearity is considered which result in nonlinear kinematic relations, while the strains are still small compared with the unity. A consequence of this is that the resulting differential equilibrium equations are coupled and nonlinear. For flat membranes the problem is less difficult and various solutions (analytical, approximate and numerical) are available in the literature [7]. For space membranes the analytical solutions are limited to axisymmetric membranes where the problem is highly simplified as it becomes one-dimensional. However, membranes of arbitrary shape encountered in realistic engineering problems can be analyzed only by numerical methods [1,8–11]. Additional related references can be found in Ref. [1] for space membranes and in Ref. [7] for flat. For the coupled cable-membrane problem the FEM has been used by several authors with various formulations. Haber et al. [12] have used a computer-aided design program for the design of cable reinforced...