On the intensity of linear elastic high order singularities ahead of cracks and re-entrant corners

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The paper deals with high order elastic singular terms at cracks and re-entrant corners (sharp V-notches), which are commonly omitted in linear elastic analyses by the argument that the strain energy and displacements in the near-tip region should be bounded. The present analysis proves that these terms are fully included in the elastic part of complete elastic–plastic stress and strain solutions.

The intensities of high order singular terms are found to be linked to the linear elastic stress intensity factor and the extension of the plastic zone along the crack bisection line. The smaller the plastic radius, the smaller the intensities of high order singular terms are.

A physical justification of the existence of high order singular terms is provided on the basis of the strain energy density distribution detected along the crack bisection line. Finally, the influence of the V-notch opening angle is made explicit, discussing also the relationship between the singularity orders and the solution of a Williams’ type sinusoidal eigen-equation.

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1. Introduction

As is well known, the linear elastic singularity order of crack tip stress fields is $-1/2$ for all the three principal loading modes. In general terms, crack tip stresses can be determined on the basis of Eqs. (1) and (2) (see Rice, 1968, and references reported therein), which give:

$$\tau_{xy} + i\tau_{zx} = z^{-1/2}h(z) + ik(z),$$

for mode III loading conditions and

$$\sigma_{xx} + \sigma_{yy} = 4Re\{z^{-1/2}f(z) + g(z)\},$$

$$\sigma_{yy} - 2i\tau_{xy} = -4iz^{-1/2}Im\{f(z)\} - 4Re\{g(z)\}$$

for mode I and mode II loading conditions. Here $h(z)$, $k(z)$, $f(z)$ and $g(z)$ are analytical functions in the vicinity of the crack tip and real function on the x axis, with x being coincident with the crack bisection.

In Eqs. (1) and (2) the crack tip stress fields exhibit inverse square root singularities, the magnitude of singular terms being determined by the value of $f(z)$ and of $h(z)$ at the origin for in-plane stresses and out-of-plane stresses, respectively. This magnitude of stress fields is universally linked, after Irwin, to the stress intensity factor (Irwin, 1957).

The functions $h(z)$, $k(z)$, $f(z)$ and $g(z)$ can be given in a polynomial form of the kind $\sum_{k=0}^{\infty}A_kz^k$, where coefficients $A_k$ can be determined by matching prescribed stresses at an appropriate number of discrete points on the boundary, as done by Gross et al. (1964) and Gross and Srawley (1965); this procedure is commonly named “Boundary collocation of stress function”.

The complex variable formulation according to Eqs. (1) and (2) leads to crack tip stresses of the kind:

$$\sigma_{ij} = \sum_{k=1}^{\infty}a_kr^{k/2-1}f_i(\theta),$$

which is consistent with the series expansion proposed by Williams for the plane problem of sharp, zero radius, V-notches (Williams, 1957).

Starting from these bases, Larsson and Carlsson (1973) performed a number of finite element analyses of cracks in various specimen geometries and found significant variations for the plastic zone size at the same value of the stress intensity factor $K$. They explained their results by recognizing the influence on the plastic region of the first nonsingular term given by Eq. (3), that proportional to $r^{1/2}$. Beyond the term proportional to $r^{-1/2}$, the constant tension was called $T$-stress for plane problems. Succeeding terms, which are nonsingular, tend naturally to zero as the crack tip is approached.