Spread Option Pricing with Finite Liquidity

A. Shidfar* Kh. Paryab* A. R. Yazdanian*

*Iran University of Science and Technology
E-mail: yazdanian@iust.ac.ir
shidfar@iust.ac.ir,paryab@iust.ac.ir

Abstract

We discuss the pricing and hedging of European spread options on correlated assets when, in contrast to the standard framework and consistent with a market with imperfect liquidity, the option trader’s trading in the stock market has a direct impact on one of the stocks price. We consider a full feedback model in which price impact is fully incorporated into the model. In order to find the numerical solution of the model, we will first apply Matched Asymptotic Expansions techniques on the price impact function to expand the full nonlinear equation to combination of linear equations. Then we use the Peaceman-Rachford scheme as an alternating direction implicit method to solve the linear equations numerically. Finally we provide a numerical analysis of the effect of the illiquidity in the underlying asset market on the replication of a European option, such that compared to the Black-Scholes case, a trader generally buys more stock to replicate a option.

Keywords and phrases: Spread option, Price impact, Illiquid markets, Nonlinear finance, Asymptotic analysis, Peaceman-Rachford scheme.

1. Introduction

Classical asset pricing theory assumes that traders act as price takers, that is, the theory assumes that investors trades have no impact on the prices paid or received. The relaxation of this assumption and its impact on realized returns in asset pricing models is called liquidity risk. Consistent with this discussion, most of the option pricing models assume that an option trader cannot affect the price in trading the underlying asset to replicate the option payoff, regardless of her trading size. The papers of Black and Scholes [1], and much of the work undertaken in mathematical finance has been aimed at relaxing of this underlying assumption. This is reasonable only in a perfectly liquid market.

In presence of such a price impact, the most important issues is how the impact price can affects the replication of an option. In order to respond to this question, there are two step. First, whether an option is still perfectly replicable or not. Second, how the presence of impact price change the replicating costs. This encouraged researchers to develop the Black-Scholes model to models that involve the price impact due to a large trader who is able to move the price by his/her actions. An excellent survey of these research can be found in [2].

Most of the research works discussed how the impact price affects the replication of an option, the our purpose of this paper is to investigate the effects of imperfect liquidity on the replication of an European Spread option by a typical option trader. The hedger is assumed