A finite iterative algorithm for solving the generalized \((P, Q)\)-reflexive solution of the linear systems of matrix equations

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A B S T R A C T

In this paper, we proposed an algorithm for solving the linear systems of matrix equations
\[
\sum_{i=1}^{N} A_i x_i = C, \quad \text{over the generalized } (P, Q)\text{-reflexive matrix } X_i \in R^{m \times n} (A_i^{(0)} \in R^{n \times m}, B_i^{(0)} \in R^{m \times q}, C_i^{(0)} \in R^{q \times p}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, M).
\]
According to the algorithm, the solvability of the problem can be determined automatically. When the problem is consistent over the generalized \((P, Q)\)-reflexive matrix \(X_i\) \((i = 1, \ldots, N)\), for any generalized \((P, Q)\)-reflexive initial iterative matrices \(X_i(0)\) \((i = 1, \ldots, N)\), the generalized \((P, Q)\)-reflexive solution can be obtained within finite iterative steps in the absence of roundoff errors. The unique least-norm generalized \((P, Q)\)-reflexive solution can also be derived when the appropriate initial iterative matrices are chosen. A sufficient and necessary condition for which the linear systems of matrix equations is inconsistent is given. Furthermore, the optimal approximate solution for a group of given matrices \(Y_i\) \((i = 1, \ldots, N)\) can be derived by finding the least-norm generalized \((P, Q)\)-reflexive solution of a new corresponding linear system of matrix equations. Finally, we present a numerical example to verify the theoretical results of this paper.

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1. Introduction

Throughout the paper, \(R^n\) will denote the complex \(n\)-vector space and the set of \(n \times m\) matrices by \(R^{n \times m}\). For a matrix \(A \in R^{n \times n}\), \(\|A\|\) represents its Frobenius norm, \(R(A)\) represents its column space, \(tr(A)\) represents its trace and \(vec(\cdot)\) represents the vec operator, i.e., \(vec(A) = (a_1^T, a_2^T, \ldots, a_n^T)^T\) for the matrix \(A = (a_1, a_2, \ldots, a_n) \in R^{n \times m}\), \(a_i \in R^m\), \(i = 1, 2, \ldots, n\). \(A \otimes B\) stands for the Kronecker product of matrices \(A\) and \(B\). In [1], the definition and some properties of generalized reflexive (anti-reflexive) matrix have been presented.

In [2], Peng and Hu presented the conditions for the solvability of matrix equation \(AX = B\) over reflexive or anti-reflexive matrices and the conditions for the solvability of matrix equation \(AXB = C\) over reflexive matrices have been presented in [3]. The sufficient and necessary conditions for the solvability of matrix equation \(A^{(0)}XB = C\) over reflexive or anti-reflexive matrices were provided in [4]. By using the generalized singular value decomposition, a necessary and sufficient condition for the matrix equation \(AXB = D\) over generalized reflexive matrices was given and the solution set was constructed explicitly when it is nonempty in [5]. In [6], the authors considered the generalized reflexive solutions for a class of matrix equations