Solution of multiple crack problem in a finite plate using an alternating method based on two kinds of integral equation

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Abstract

This paper investigates a solution of multiple crack problem in a finite plate using an alternating method. The finite plate with cracks is an overlapping region of two regions: namely the infinite region exterior to the cracks and the finite region interior to finite plate without cracks. It is assumed that the cracks are applied by some loading and edges of the finite plate are of traction free. Governing equations for the problem and an alternating method are suggested. In the iteration, we need to solve two boundary value problems. One is the multiple crack problem in an infinite plate, and the other is the boundary value problem for the finite plate without crack. Several numerical examples are provided to prove the effectiveness of the suggested method.

1. Introduction

Schwarz proposed an iterative method for the solution of classical boundary value problems for harmonic functions. Schwarz’s alternating method is also used to solve boundary value problems (BVP) of partial differential equations of elliptic type [1]. An alternating method for analyzing the interactions among multiple circular holes in a two-dimensional infinite domain was proposed [2]. A boundary spectral method for elastostatic problems with multiple spherical cavities and inclusions was studied. The studied problem with multiple spherical features is solved by using Schwarz’s alternating method [3]. For a sectorial domain, the Schwarz alternating methods with overlapping and non-overlapping domain were proposed for the Laplace equation [4]. A Schwarz’s alternating algorithm for elliptic boundary value problems in an infinite domain with a concave angle was investigated, which belongs to an overlapping domain decomposition method [5]. A discrete technique of the Schwarz alternating procedure is presented to combine the Ritz–Galerkin and finite element methods [6].

This paper investigates a solution of multiple crack problem in a finite plate using an alternating method. The finite plate with cracks is an overlapping region of two regions: namely the infinite region exterior to the cracks and the finite region interior to finite plate without crack. A Fredholm integral equation is used to solve the multiple crack problem in an infinite plate, and the unusual boundary integral equation is suggested to solve the boundary value problem of the finite plate. The interactions of two kinds of the integral equation are exactly evaluated. This means that the traction influences on the crack faces from cracks themselves and from the boundary traction are coupled. Similarly, the traction influences on the boundary from boundary itself and from tractions on crack faces are also coupled. The coupled effects, or the interactions between two kinds of integral equation, are solved numerically by using an alternating method. Since two kinds of integral equation are used in the iteration, the suggested method is not exactly the Schwarz alternating method.

In the iteration, we need to solve two boundary value problems successively. One is the multiple crack problem in an infinite plate, and the other is the boundary value problem for the finite plate without crack. For the multiple crack problem, it can be solved very well based on many recent publications [7,8]. For the interior boundary problem for a finite plate, a complex variable formulation of boundary integral equation is suggested [9]. Stress intensity factors and T-stresses are investigated in the formulation. Several numerical examples are provided to prove the effectiveness of the suggested method.

2. Analysis

2.1. Basic concept for the solution of a crack problem in a finite plate using an alternating method

The basic concept for the solution of a crack problem in a finite plate using an alternating method is introduced below. It is