A functional generalization of the reverse Hölder integral inequality on time scales

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\section*{ABSTRACT}

In this paper, we establish a functional generalization of the diamond-$\alpha$ integral reverse Hölder inequality on time scales. Some related inequalities are also considered.

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\section{1. Introduction}

Let $1/p + 1/q = 1$. Assume that $f(x)$ and $g(x)$ are continuous real-valued functions on $[a, b]$. Then

(1) for $p > 1$, we have the following Hölder inequality (see [1]):

$$
\int_a^b f(x)g(x)dx \leq \left( \int_a^b f^p(x)dx \right)^{1/p} \left( \int_a^b g^q(x)dx \right)^{1/q}.
$$

(1.1)

(2) for $0 < p < 1$, we have the following reverse Hölder inequality (see [2]):

$$
\int_a^b f(x)g(x)dx \geq \left( \int_a^b f^p(x)dx \right)^{1/p} \left( \int_a^b g^q(x)dx \right)^{1/q}.
$$

(1.2)

The above inequalities play an important role in many areas of pure and applied mathematics. A large number of generalizations, refinements, variations and applications of (1.1) and (1.2) have been investigated in the literature (see [3–11] and the references therein). Recently, Yang [12] gave a functional generalization of (1.1) on time scales.

The aim of this paper is to give a functional generalization of the diamond-$\alpha$ integral reverse Hölder inequality on time scales. Some related inequalities are also considered. Due to restrictions on the number of pages, the basic definitions and theorems of time scale calculus are omitted, and the reader is referred to [13–16].

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