Dynamic element discretization method for solving 2D traction boundary integral equations

Z.R. Jin\textsuperscript{a}, D.M. Zhang\textsuperscript{b}, Y.H. Wang\textsuperscript{b,*}, Z.L. Li\textsuperscript{b}

\textsuperscript{a} School of Civil Engineering, Hunan City University, Yiyang, Hunan 413000, China
\textsuperscript{b} School of Civil Engineering & Mechanics, Huazhong University of Science & Technology, Wuhan, Hubei 430074, China

\textbf{ABSTRACT}

A sufficient condition for the existence of element singular integral of the traction boundary integral equation for elastic problems requires that the tangential derivatives of the boundary displacements are Hölder continuous at collocation points. This condition is violated if a collocation point is at the junction between two standard conforming boundary elements even if the field variables themselves are Hölder continuous there. Various methods are proposed to overcome this difficulty. Some of them are rather complicated and others are too different from the conventional boundary element method. A dynamic element discretization method to overcome this difficulty is proposed in this work. This method is novel and very simple: the form of the standard traction boundary integral equation remains the same; the standard conforming isoparametric elements are still used and all collocation points are located in the interior of elements where the continuity requirements are satisfied. For boundary elements with boundary points where the field variables themselves are singular, such as crack tips, corners and other boundary points where the stress tensors are not unique, a general procedure to construct special elements has been developed in this paper. Highly accurate numerical results for various typical examples have been obtained.

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1. Introduction

Two-dimensional elastic boundary-value problems with various kinds of singular boundary conditions will be investigated by boundary integral equations (BIEs) using boundary element methods (BEMs) in this paper. The BIE formulations and the shape functions used in BEM may produce great effect on the accuracy of the numerical computations. For simplicity, let \( u_i \) and \( t_i \) designate boundary displacements, tangential derivatives of boundary displacements and boundary tractions of an elastic body, respectively. It is well known that \( u_i \) is continuous for all boundary points. However, \( u_i \) and \( t_i \) at some boundary points, which are called singular boundary points in this paper, may have various types of singularities, such as \( \ln r \) weak singularity, discontinuity of the first kind and singularity of \( r^{-\lambda} \) (\( 0 < \lambda < 1 \)). If a smooth boundary point is singular, the stress tensor at that point is multi-valued and/or un-bounded. If stress tensors are unique for all boundary points, very good BEM numerical results can be obtained using the displacement-BIE with isoparametric elements. On the other hand, traction-BIE is more suitable to treat problems with singular boundary points, in some cases (e.g., crack problems) where the use of traction-BIE appeared unavoidable.

The integral kernel related to \( u_i \) of a displacement-BIE has Cauchy singularity. It is well known that a sufficient condition for the existence of Cauchy principal-value integral is that \( u_i \) is \( C^{0,\alpha} \) continuous. This condition is always satisfied for elasticity problems. The integral kernel related to \( u_i \) of a traction-BIE has Hadamard hyper-singularity, a sufficient condition for the existence of Hadamard finite-part integral is that \( u_i \) is \( C^{1,\alpha} \) continuous \([1,2]\). The degenerate traction-BIE with \( u_i \) and \( t_i \) as field variables will be used in this paper. Both integral kernels related to \( u_i \) and \( t_i \) of the degenerate traction-BIE have Cauchy singularity, the sufficient condition for the existence of the singular integrals requires \( C^{0,\alpha} \) continuity for \( u_i \) and \( t_i \). Therefore, the sufficient condition for the existence of the singular integrals still requires \( C^{1,\alpha} \) continuity for \( u_i \). A weaker condition for the existence of singular integrals of various kinds of BIEs has been provided in \([2]\).

In traditional BEM, piecewise continuous shape functions composed of the Lagrangian polynomials are often used to approximately describe the geometry of elements where the