On torsion of a single crystal rod

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The paper aims at calculating the dislocation distribution inside a single crystal rod loaded
in torsion within the framework of continuum dislocation theory. We construct an explicit
analytical solution of this problem in terms of the modified Bessel and hypergeometric
functions. The interesting features of this solution are the energetic and dissipative thresh-
olds for dislocation nucleation, the translational work hardening, and the size effect. The
comparison with experimental results shows quite good agreement of the torque-twist
curves for small up to moderate twists.

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1. Introduction

Mechanical responses of engineering materials are characterized by the materials’ microstructure given in terms of the
spatial distribution of all structural features on the microscale. Microstructural characteristics, besides those of the periodic
crystal lattice which determine the elastic properties, comprise point defects, dislocations, grain boundaries, solute particles
or phase boundaries, to mention only a few. Plastic deformation of crystalline materials depends to a high degree on the
mechanisms related to the dislocation network: in order to accommodate plastic distortion and to reduce the crystal’s
energy, new dislocations are nucleated and pile up ahead of the grain or phase boundaries, thereby giving rise to material
strengthening. The nucleation and motion of dislocations is hence a crucial mechanism to explaining plastic yielding, work
hardening and hysteresis in crystal plasticity. Although dislocations appear in the crystal lattice of discrete nature, contin-
uum descriptions of geometrically necessary dislocations were established and have been found useful as a mathematical
tool for investigating the complex behavior of the dislocation network. This continuum approach is dictated by the high dis-
location densities accompanying plastic deformations, which typically range from 10^8 to 10^14 m^-2. Although the framework
of continuum dislocation theory has been laid down long time ago by Kondo (1952), Nye (1953), Bilby et al. (1955), Kröner
(1958), Berdichevsky and Sedov (1967), and Le and Stumpf (1996a,b,c), the applicability of the theory became feasible only
in recent years (Ortiz and Repetto, 1999; Berdichevsky, 2006a) thanks to the progress in statistical mechanics and thermo-
dynamics of the dislocation network (Berdichevsky, 2005, 2006b). Among various alternative strain gradient plasticity the-
ories we mention here only those of Shu and Fleck (1999), Gao et al. (1999), Acharya and Bassani (2000), Huang et al. (2000,
2004), Fleck and Hutchinson (2001), Han et al. (2005a,b). All of these strain gradient plasticity theories have as a common
feature the incorporation of plastic strain gradient into the energy, thus leading to its dependence on the history of plastic
deformation, in contrast to the continuum dislocation theory. Gurtin and co-workers (Gurtin, 2003, 2004; Gurtin and Anand,
2005) used second powers of plastic strain gradients in their works to incorporate the defect energy or the energy of the
microstructure, see also (Volokh and Trapper, 2007). A crucial question is how to formulate the contribution of the