Boundary element modeling of 3D anisotropic heat conduction involving arbitrary volume heat source

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ABSTRACT

This article presents a BIE (Boundary Integral Equation) modeling that applies the technique of domain mapping (Shiah and Tan (1998) [12]) as well as MRM (Multiple Reciprocity Method (Nowak (1989) [10])) to treat the problem of 3D anisotropic heat conduction involving arbitrary volume heat source. By the domain mapping technique, the original function of the volume heat source is accordingly transformed to a new function defined in the mapped domain. As a result of applying MRM, the volume integral arising from the transformed heat source is transformed into a series of boundary integrals. At the end of this paper, three numerical examples are presented to show the veracity and accuracy of the proposed algorithm. Also, the convergence of the transformed boundary integrals is also investigated for different types of volume heat source functions.

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1. Introduction

In engineering practice, various heat conduction problems often involve internal volume heat source that may be described by a complicated function in anisotropic media due to internal chemical reactions or electrical heating. Mathematically, the problem is governed by an elliptical partial differential equation. Despite its importance in engineering applications, relatively scarce papers in this regard appear in the open literature. To name a few as example, Zyong [1] solved a system of stationary equations of heat conduction and filtration of molten material in the presence of a volume heat source generated by absorption of the energy of electromagnetic radiation. Jayaram and Strieder [2] analyzed the melting of a semi-infinite slab initially at its melting temperature with a volume heat source, distributed within a supporting substrate layer by the heat balance integral method. For engineering analyses, the most popular computer methods include, for example, the finite difference method, the finite element method, and the boundary element method (BEM). The former two, used frequently in numerical modeling, are referred to as domain solution techniques that require full discretization of the whole domain.

In recent years, BEM has been recognized as an efficient numerical tool for modeling engineering problems due to its distinctive feature that only the boundary needs to be modeled. However, in BEM’s direct formulation for heat conduction problems, the internal volume heat source reveals itself as a volume integral, which conventionally demands internal cell discretization throughout the whole domain and therefore destroys the BEM’s notion as a boundary solution technique. By generalizing the Galerkin Vector Approach, the Multiple Reciprocity Method (MRM), introduced by Nowak [3], employs a set of higher-order fundamental solutions that permits the application of Green’s identity to each term of the sequence in succession. For solving the steady state heat conduction problem, Chen and Hong [4] have introduced the theory of dual integral equations that reveal different orders of singularities in kernels. As a matter of fact, MRM is nothing more than the real-part BEM in a series form [5]. Also, Chen and Wong [6] have studied the MRM in presenting the dual formulations...