Nonlinear 3D numerical computations for the square membrane versus experimental data

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This paper presents a discussion on the three-dimensional (3D) mechanical model proposed by Shi et al. for the membrane deformation by comparing the experimental results, and the limitation of this model to predict the behaviors of the membrane structure with large deformations (i.e. geometric nonlinearity). Three nonlinear numerical models, all of which can avoid the limitation, are then established to simulate the membrane’s large deformations, including the membrane model with zero bending stiffness and the shell model with small but nonzero bending stiffness based on the latest Edge-based Smoothed Finite Element Method (ES-FEM) as well as the standard Finite Element Method (FEM). The effects of geometric nonlinearity on the numerical results are carefully checked by comparing the benchmark experimental results, and the effects of different models/methods on the numerical results are also quantitatively examined. Factors, e.g. pressure fluctuations in the experiment and boundary conditions in the numerical models, are discussed to illustrate the differences between the numerical and experimental results, so as to provide some further suggestions on the improvements of the corresponding numerical models.

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1. Introduction

Membrane structures are widely used in huge areas, e.g. coverings in architecture, diaphragms in transducers, artificial arteries and organs in biomedical prostheses, and radio antennas and optical reflectors in airships [1,2]. One major reason for such a wide use is their lightweight, flexibility and being highly susceptible to external action [3].

In architecture various shapes of membrane structures can be formulated by simply fastening them to stiff continuous substrates or discrete supports, in order to achieve different design purposes. Ballooning always occurs to these fastened membranes in one out-of-plane direction due to the negative air pressure differential (usually caused by wind) [4], which may induce excessive stretch, rupture and tear in the membranes, and fatigue at the fasteners’ deck engagement location [4–7]. Therefore, accurately predicting the ballooning shapes of the fastened membranes is essential during the design process to ensure the safety of the structures. Theoretically determining the ballooning shapes of the membranes is very difficult, because finding the solutions of the differential equations governing the flexible membrane is not an easy task. Analytical solutions are only available for homogeneous isotropic membranes with very simple geometry and loading conditions, such as a circular membrane under axisymmetric loading, where the problem becomes one-dimensional [1].

In order to determine the ballooning shapes of the complex membranes, simplified models are often built [8–11]. Shi et al. created two models to estimate the deformations of the square membranes, one of which is a two-dimensional (2D) model with the two opposite edges being continuously constrained [11], and the other is a three-dimensional (3D) model with all the four vertices being fully constrained [4]. In these two models, the deformed profiles of the membranes were both assumed to be governed by parabolic equations, and the requirements of the force equilibrium, deformation compatibility and stress–strain relationship were all satisfied during deriving these governing equations. Baskaran et al. compared the predicted maximum deflections from the 2D model [11] with those from a series of experiments, and concluded that this 2D model always underestimates the maximum deflections [5]. However, the reasons for the underestimation have not been detailed. Furthermore, the 3D model [4] has not been examined and verified yet.

Numerical simulation is another efficient way to analyze the deformations of membranes with complex profiles, and the standard Finite Element Method (FEM) is frequently used to determine the ballooning shapes of these deformed membranes [12–14].