Magneto elastic stress subjected to uniform magnetic field over a thin infinite plate containing an elliptical hole with an edge crack

Norio Hasebe *

Department of Civil Engineering, Nagoya Institute of Technology, Gokiso-cho Showa-ku, Nagoya 466-8555, Japan

Nagoya Industrial Science Research Institute, Yotsuyadori 1-13, Chikusa-ku, Nagoya 464-0819, Japan

A R T I C L E   I N F O

Article history:
Received 30 October 2010
Received in revised form 15 March 2011
Available online 23 March 2011

Keywords:
Magneto stress
Maxwell stress
Shear deflection
Magnetic field
Soft ferromagnetic material
Paramagnetic material
Elliptical hole
Edge crack
Stress intensity factor
Mapping function

A B S T R A C T

Two dimensional solutions of the magnetic field and magneto elastic stress are presented for a magnetic material of a thin infinite plate containing an elliptical hole with an edge crack subjected to uniform magnetic field. Using a rational mapping function, each solution is obtained as a closed form. The linear constitutive equation is used for these analyses. According to the electro-magneto theory, only Maxwell stress is caused as a body force in a plate. In the present paper, it raises a plane stress state for a thin plate, the deformation of the plate thickness and the shear deflection. Therefore the magneto elastic stress is analyzed using Maxwell stress. No further assumption of the plane stress state that the plate is thin is made for the stress analysis, though Maxwell stress components are expressed by nonlinear terms. The rigorous boundary condition expressed by Maxwell stress components is completely satisfied without any linear assumptions on the boundary. First, magnetic field and stress analyses for soft ferromagnetic material are carried out and then those analyses for paramagnetic and diamagnetic materials are carried out. It is stated that those plane stress components are expressed by the same expressions for those materials and the difference is only the magnitude of the permeability, though the magnetic fields $H_x, H_y$ are different each other in the plates. If the analysis of magnetic field of paramagnetic material is easier than that of soft ferromagnetic material, the stress analysis may be carried out using the magnetic field for paramagnetic material to analyze the stress field, and the results may be applied for a soft ferromagnetic material. It is stated that the stress state for the magnetic field $H_x, H_y$ is the same as the pure shear stress state. Solutions of the magneto elastic stress are nonlinear for the direction of uniform magnetic field. Stresses in the direction of the plate thickness and shear deflection are caused and the solutions are also obtained. Figures of the magnetic field and stress distribution are shown. Stress intensity factors are also derived and investigated for the crack length.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

In the present paper, an infinite plate containing an elliptical hole with an edge crack exposed by magnetic field from out side of the plate are analyzed. In the previous paper [Hasebe, 2010a], the plate applied magnetic field induced by electric current was analyzed. Uniform magnetic field is one of the typical magnetic field as well as magnetic field caused by electric current. Main reviews have been stated in the previous paper; therefore the reviews are omitted for the limit pages.

Maxwell equations are essentially three dimensional ones for the magnetic field. Generally speaking, three dimensional boundary value problems are more difficult than those of two dimensional ones; therefore many problems have been modeled and analyzed as two dimensional problems. It seems not to be easy to make the two dimensional model of the magnetic field. However, when the plate is thin, the magnetic field in the plate with a hole can be obtained; therefore, the analysis is carried out for the thin plate, and also plane stress analysis can be applied.

According to the electro-magneto theory, only Maxwell stress components are caused as the body force in the magnetic material; therefore, Maxwell stress is considered for the stress analysis. No further assumptions for the magnetic stress analysis are made except the assumption of the plane stress state that the plate is thin, though Maxwell stress components and the boundary condition are expressed by the nonlinear terms of Maxwell stress components.

Intensities of the magnetic field component and stress intensity factors at the crack tip are obtained. The relationships among paramagnetic, diamagnetic and soft ferromagnetic materials are investigated for magneto elastic stress. Also the present problem causes shear deflection and the stress in the direction of the plate...