Fuzzy \*-homomorphisms and fuzzy \*-derivations in induced fuzzy \(C^*\)-algebras

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**ABSTRACT**

Using the fixed point method, we prove the Hyers–Ulam stability of the Cauchy–Jensen functional equation and of the Cauchy–Jensen functional inequality in fuzzy Banach \*-algebras and in induced fuzzy \(C^*\)-algebras.

Furthermore, using the fixed point method, we prove the Hyers–Ulam stability of fuzzy \*-derivations in fuzzy Banach \*-algebras and in induced fuzzy \(C^*\)-algebras.

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1. Introduction and preliminaries

The theory of fuzzy space has progressed greatly, developing the theory of randomness. Some mathematicians have defined fuzzy norms on a vector space from various points of view \[1–6\]. Following Cheng and Mordeson \[7\], Bag and Samanta \[1\] gave an idea of fuzzy norm in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type \[8\] and investigated some properties of fuzzy normed spaces \[9\].

We use the definition of fuzzy normed spaces given in \[1,5,10\] to investigate a fuzzy version of the Hyers–Ulam stability for the Cauchy–Jensen functional equation in the fuzzy normed \*-algebra setting.

**Definition 1.1** \(\{1,5,10,11\}\). Let \(X\) be a complex vector space. A function \(N : X \times \mathbb{R} \rightarrow [0, 1]\) is called a fuzzy norm on \(X\) if for all \(x, y \in X\) and all \(s, t \in \mathbb{R}\),

\[
\begin{align*}
(N_1) & \quad N(x, t) = 0 \text{ for } t \leq 0; \\
(N_2) & \quad x = 0 \text{ if and only if } N(x, t) = 1 \text{ for all } t > 0; \\
(N_3) & \quad N(cx, t) = N(x, \frac{t}{|c|}) \text{ if } c \in \mathbb{C} \setminus \{0\}; \\
(N_4) & \quad N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}; \\
(N_5) & \quad N(x, \cdot) \text{ is a non-decreasing function of } \mathbb{R} \text{ and } \lim_{t \to \infty} N(x, t) = 1; \\
(N_6) & \quad \text{for } x \neq 0, N(x, \cdot) \text{ is continuous on } \mathbb{R}.
\end{align*}
\]

The pair \((X, N)\) is called a fuzzy normed vector space.

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