A combined analytical, numerical, and experimental study of shape-memory-alloy helical springs

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**Abstract**

In this paper, the pseudoelastic response of shape memory alloy (SMA) helical springs under axial force is studied both analytically and numerically. In the analytical solution two different approximations are considered. In the first approximation, both the curvature and pitch effects are assumed to be negligible. This is the case for helical springs with large ratios of mean coil radius to the cross sectional radius (spring index) and small pitch angles. Using this assumption, analysis of the helical spring is reduced to that of the pure torsion of a straight bar with circular cross section. A three-dimensional phenomenological macroscopic constitutive model for polycrystalline SMAs is reduced to the one-dimensional pure shear case and a closed-form solution for torsional response of SMA bars in loading and unloading is obtained. In the next step, the curvature effect is included and the SMA helical spring is analyzed using the exact solution presented for torsion of curved SMA bars. In this refined solution, the effect of the direct shear force is also considered. In the numerical analyses, the three-dimensional constitutive equations are implemented in a finite element method and using solid elements the loading–unloading of an SMA helical spring is simulated. Analytical and numerical results are compared and it is shown that the solution based on the SMA curved bar torsion gives an accurate stress analysis in the cross section of the helical SMA spring in addition to the global load–deflection response. All the results are compared with experimental data for a Nitinol helical spring. Several case studies are presented using the proposed analytical and numerical solutions and the effect of changing different parameters such as the material properties and temperature on the loading–unloading hysteretic response of SMA helical springs is studied. Finally, some practical recommendations are given for improving the performance of SMA helical springs used as energy dissipating devices, for example for seismic applications.

1. Introduction

Since the first observation of the shape memory effect (SME) in some alloys (Chang and Read, 1951a,b), studying shape memory alloys (SMAs) has been an active field of research. The unique ability of shape memory alloys in recovering large inelastic strains and also generating high stresses has caused a considerable increase in manufacturing devices made of these materials in recent years. The SMAs are now used in applications in a wide variety of devices ranging from simple parts like cell phone antennas or eyeglass frames to complicated devices in mechanical (Brook, 1983; Jee et al., 2006; Xua and Song, 2004), biomechanical (Pettrini et al., 2005), aerospace (Hartl and Lagoudas, 2007), and civil engineering (DesRoches and Smith, 2004).

The unique macroscopic properties of SMAs are based on the solid–solid phase transition of the underlying lattice between a high symmetry cubic lattice (austenite) and a low symmetry lattice (martensite). It is known that when the SMA atoms are arranged in the cubic austenite lattice form, the entropy and internal energy are higher compare to the martensite lattice. The competition between the entropy and internal energy is reflected in the free energy $F = U - TS$, where $U$ is the internal energy, $S$ is the entropy and $T$ is temperature. It is known that at higher temperatures the entropy overcomes the competition and the austenite phase is preferred while at lower temperatures the internal energy determines the stability and the martensite phase is preferred (Kastner, 2003, 2006). For a comprehensive discussion on general properties of SMAs and the phase transformation phenomenon, readers are referred to Müller and Xu (1991) and Müller and Seelecke (2001).

As a result of the solid–solid phase transformation (usually called martensitic phase transformation), and according to the specific way the transformation occurs, SMAs exhibit two significant macroscopic phenomena: the shape memory effect and pseudoelasticity. Each of these two macroscopic responses to mechanical and/or thermal loading is the origin of a vast range of applications.