Interactive fuzzy random two-level linear programming through fractile criterion optimization

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ABSTRACT

In this paper, assuming cooperative behavior of the decision makers, solution methods for decision making problems in hierarchical organizations under fuzzy random environments are considered. To deal with the formulated two-level linear programming problems involving fuzzy random variables, $\alpha$-level sets of fuzzy random variables are introduced and an $\alpha$-stochastic two-level linear programming problem is defined for guaranteeing the degree of realization of the problem. Taking into account the vagueness of judgments of decision makers, fuzzy goals are introduced and the $\alpha$-stochastic two-level linear programming problem is transformed into the problem to maximize the satisfaction degree for each fuzzy goal. Through the use of the fractile criterion optimization model, the transformed stochastic two-level programming problem can be reduced to a deterministic one. Interactive fuzzy programming to obtain a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented. It is shown that all of the problems to be solved in the proposed interactive fuzzy programming can be easily solved by the simplex method, the sequential quadratic programming or the combined use of the bisection method and the sequential quadratic programming. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method.

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1. Introduction

Fuzzy random variables, first introduced by Kwakernaak [1], have been developing in various ways [2–4]. An overview of the developments of fuzzy random variables was found in the article of Gil et al. [5]. Studies on linear programming problems with fuzzy random variable coefficients, called fuzzy random linear programming problems, were initiated by Wang and Qiao [6], Qiao et al. [7] as seeking the probability distribution of the optimal solution and optimal value. Optimization models for fuzzy random linear programming problems were first considered by Luhandjula et al. [8,9] and further developed by Liu [10,11] and Rommelfanger [12]. A brief survey of major fuzzy stochastic programming models was found in the paper by Luhandjula [13]. As we look at recent developments in the fields of fuzzy random programming, we can see continuing advances [14–19,12,20–22].

However, decision making problems in hierarchical managerial or public organizations are often formulated as two-level mathematical programming problems [23]. In the context of two-level programming, the decision maker at the upper level first specifies a strategy, and then the decision maker at the lower level specifies a strategy so as to optimize the objective with full knowledge of the action of the decision maker at the upper level. In conventional multi-level mathematical programming models employing the solution concept of Stackelberg equilibrium, it is assumed that there

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