Two classical transportation problems revisited: Pure constant fixed charges and the paradox

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We analyze degeneracy characterizations for two classical problems: the transportation paradox in linear transportation problems and the pure constant fixed charge transportation problem. Solving the pure constant fixed charge problem is equivalent to finding a basic tree solution with maximum degree of degeneracy. Problems possess degenerate solutions if the equalsubsum property is satisfied for the supplies and demands. Determining the existence of degeneracy is an NP-complete problem. But this NP-hardness remains even if all equalsubsums are known in advance. For the second problem, the transportation paradox, there exists a vast literature that typically describes methods, derived within the framework of the classical transportation algorithm, for determining solutions where the more-for-less phenomenon occurs. We show how to solve this problem as a simple standard network flow problem. The paradox is linked to overshipments solutions, which belong to supply and demand configurations that tend to have a high degree of degeneracy.

1. Introduction

The classical linear transportation problem (TP) introduced originally in [1] may be described as follows. A commodity is to be transported from each of m sources to each of n destinations represented as a bipartite graph. The supplies available at each of the sources are aᵢ, i = 1, . . . , m; and the demands at destinations are bⱼ, j = 1, . . . , n respectively. It is assumed that \( \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \), i.e. the total supply is equal to the total demand. The linear cost of transporting one unit of the commodity from supply node i to destination node j is cᵢⱼ; this cost is generally given in a cost matrix \( C = (cᵢⱼ) \); see Fig. 1. We denote by \( P(a, b, C) \) an instance of the TP. The goal is to determine the amounts \( xᵢⱼ \) to be transported over all routes (i, j) such that the total transportation cost is minimized. The classical transportation problem is a special case of the problem of max flow and min cost and is therefore polynomial. The mathematical formulation as a linear program is given in Fig. 2.

The so-called transportation paradox or more-for-less paradox in transportation problems occurs when it is possible to ship more total goods for less total cost, while shipping more or equal amounts from each origin and to each destination. This phenomenon was discovered by Charnes and Klingman [2] and Szwarc [3]. The source of this phenomenon is intuitively explained in [4]. An exact characterization of all cost matrices \( C = (cᵢⱼ) \) that are immune against the paradox was given in [5].

A necessary condition is already given in [3] in terms of the dual variables \( u_i \) and \( v_j \) of the transportation problem:

No paradox occurs if \( u_i + v_j \geq 0 \) for all i and j.

Therefore the interesting positions are for the indexes p and q satisfying \( u_p + v_q < 0 \) (see also [6–10]). This is only a necessary condition (there may be no paradox in the presence of degeneracy). The problem is now how to proceed