Blow-up solutions of nonlinear Volterra integro-differential equations

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Abstract
The paper studies the finite-time blow-up theory for a class of nonlinear Volterra integro-differential equations. The conditions for the occurrence of finite-time blow-up for nonlinear Volterra integro-differential equations are provided. Moreover, the finite-time blow-up theory for nonlinear partial Volterra integro-differential equations with general kernels is also established using the blow-up results for the nonlinear Volterra integro-differential equations.

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1. Introduction
The theory of finite-time blow-up solutions has been well studied for nonlinear Volterra integral equations (see [1–5]). However, to the best of our knowledge, there are no analogous results for nonlinear Volterra integro-differential equations (VIDEs). In this paper, we study the finite-time blow-up theory for nonlinear VIDEs and nonlinear partial Volterra integro-differential equations (PVIDEs).

In the first part of this paper, we study VIDEs of the form:

\[ y'(t) = -ay(t) + \int_0^t k(t-s)g(y(s)) \, ds, \quad t > 0, \tag{1} \]

with \( y(0) = y_0 \geq 0 \), where we assume that \( a \) is a nonnegative constant and

(a) \( k(t) \) is an integrable positive function such that \( \lim_{t \to \infty} K(t) = \infty \), where \( K(t) = \int_0^t k(s) \, ds \),

(b) \( g(t) \) is nonnegative, nondecreasing and continuous for \( t > 0 \), \( g \equiv 0 \) for \( t \leq 0 \), and

\[ \lim_{y \to \infty} \frac{g(y)}{y} = \infty. \]

The finite-time blow-up theories for Eq. (2) and more general types of nonlinear Volterra integral equations are established in [1–6]. We know that Brunner and Yang [6] have recently developed a new technique to investigate the blow-up theories for VIEs and partially for VIDEs including Eq. (1) with \( a < 0 \). However, the case (1) with \( a > 0 \), which is more difficult, is not studied in their paper. In this paper we investigate problem (1) with \( a > 0 \).

For \( a \equiv 0 \) and \( y_0 \equiv 0 \), Eq. (1) can be easily converted into a VIE

\[ y(t) = \int_0^t h(t-s)g(y(s)) \, ds, \tag{2} \]