Variationally-based theories for buckling of partial composite beam–columns including shear and axial effects

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\textbf{A B S T R A C T}

This paper is focused on elastic stability problems of partial composite columns: the conditions for the axial load not to introduce any pre-bending effects in composite columns; the equivalence, similarities and differences between different sandwich and partial composite beam theories with and without the effect of shear, with and without the effect of axial extensibility, and also the effect of eccentric axial load application. The basic modelling of the composite beam–column uses the Euler–Bernoulli beam theory and a linear constitutive law for the slip. In the analysis of this reference model, a variational formulation is used in order to derive relevant boundary conditions. The specific loading associated with no pre-bending effects before buckling is geometrically characterized, leading to analytical buckling loads of the partial composite column. The equivalence between the Hoff theory for sandwich beam–columns, the composite action theory for beam–columns with interlayer slip and the corresponding Bickford–Reddy theory, is shown from the stability point of view. Special loading configurations including eccentric axial load applications and axial loading only on one of the sub-elements of the composite beam–column are investigated and the similarity of the behaviour to that of imperfect ordinary beam–columns is demonstrated. The effect of axial extensibility on kinematical relationships (according to the Reissner theory), is analytically quantified and compared to the classical solution of the problem. Finally, the effect of incorporating shear in the analysis of composite members using the Timoshenko theory is evaluated. By using a variational formulation, the buckling behaviour of partial composite columns is analysed with respect to both the Engesser and the Haringx theory. A simplified uniform shear theory (assuming equal shear deformations in each sub-element) for the partial composite beam–column is first presented, and then a refined differential shear theory (assuming individual shear deformations in each sub-element) is evaluated. The paper concludes with a discussion on this shear effect, the differences between the shear theories presented and when the shear effect can be neglected.

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1. Background and problem formulation

1.1. Introduction and literature review

This paper is devoted to the stability problem of a partial composite column, or equivalent composite sandwich column, and more specifically on the specific loading mode influencing the stability problem. Layered structural elements with interlayer slip are typically encountered in wood design, where wooden beams are made up of layers assembled by means of nailing, bolting or gluing (with a soft shear modulus). Partial composite structures built up by sub-elements of different materials and connected by shear connectors to form an interacting unit, such as timber–concrete or steel–concrete elements, are widely used in building engineering. In the case of a flexible connection, the analysis procedure requires consideration of the interlayer slip between the sub-elements, leading to the partial interaction concept.

For a detailed literature background on the partial composite theory, the readers are referred to Girhammar and Gopu [1] and Girhammar and Pan [2]. Möhler [3] obtained the buckling formulae of axially loaded partial composite columns, using Euler–Bernoulli model for each sub-element of the beam–column. The lateral buckling problem of partial composite beam–columns subjected to both transverse and axial loading was investigated by e.g. Girhammar and Gopu [4,5] and Girhammar and Pan [2].