Transient thermoelastic waves in an anisotropic hollow cylinder due to localized heating

Ravi Chitikireddy a, Subhendu K. Datta b,⇑, Arvind H. Shah a, Hao Bai c

a The University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2
b University of Colorado, Boulder, CO 80309-0427, USA
c Lakehead University, Thunder Bay, Ontario, Canada P7B 5E1

ABSTRACT

This paper presents a theoretical study of transient ultrasonic guided waves generated by concentrated heating of the outer surface of an infinite anisotropic hollow circular cylinder. Generalized thermoelastic theory proposed by Lord and Shulman is adopted to model the dynamic thermoelastic behavior of the cylinder. The concentrated heat source model used is to represent heating due to a pulsed laser beam, which is focused on the outer surface of the cylinder. A semi-analytical finite element (SAFE) method is employed to evaluate guided wave modes in the cylinder. Using integral transform techniques, the modal wave forms are obtained in frequency and wave number domains. Time histories of the propagating modes are then calculated by applying inverse Fourier transformation in the time domain. Numerical results showing the dispersion curves for the group velocities of the propagating modes and transient radial displacements are presented. For this purpose it is assumed that the cylinder is made of transversely isotropic silicon nitride (Si$_3$N$_4$). Attention is focused on the propagation characteristics of longitudinal and flexural modes separately.

1. Introduction

In recent years, laser based ultrasonic techniques have found wide use for material properties characterization and NDE for electronic and high temperature applications. These techniques not only overcome the difficulty of using piezoelectric transducers on curved complex surfaces but also provide a number of advantages over conventional ultrasonic methods, such as noncontact generation and detection of waves having a large bandwidth, and ability to operate away from hot and corrosive environment (see Scruby and Drain, 1990). White (1963) demonstrated that elastic waves can be generated by irradiating a surface by concentrated laser beams. Since then significant research has been done on laser ultrasonics theoretically and experimentally. Most of these works treat the problem as isothermal with the heat source replaced by equivalent surface forces (or force couples).

The classical theory of heat conduction equation is a parabolic partial differential equation, which predicts an infinite speed of thermal wave. This assumption of infinite speed of heat is contrary to physical phenomenon. To rectify this paradox, several generalized theories of thermoelasticity have been proposed. Lord and Shulman (1967) presented a generalized theory of thermoelasticity which takes into account the finite speed of thermal wave by inclusion of a thermal relaxation time. Green and Lindsay (1972) developed a temperature – rate dependent theory that included two relaxation times and Green and Naghdi (1993) introduced a thermoelasticity theory without energy dissipation.

Spicer et al. (1990) studied laser ultrasonic waves in a thin plate theoretically and experimentally to compute the elastic moduli and the plate thickness. Rayleigh–Lamb waves in plates using the Lord–Shulman theory have been discussed by Datta and Shah (2009). References to many other works can be found in this book (see also, Sharma et al., 2000; Verma, 2002; Verma and Hasebe, 2001; Al-Qahtani and Datta, 2004; Al-Qahtani et al., 2005; Xu et al., 2008).

Compared to many studies that have dealt with plates there is a very limited amount of reported work on waves in cylinders using generalized thermoelastic theories. Dispersion of longitudinal thermoelastic wave propagation in a circular isotropic cylinder was studied by Erbay and Suhubi (1986), who considered the cylinder surface to be stress – free and at a constant temperature. Elnagar and Abd-Alla (1987) studied the influence of the initial stress on the Rayleigh wave propagation in a generalized...