PSPACE Tableau Algorithms for Acyclic Modalized \( \mathcal{ALC} \)

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Abstract We study \( \mathcal{ALCK}_m \) and \( \mathcal{ALCS}_4^m \), which extend the description logic \( \mathcal{ALC} \) by adding modal operators of the basic multi-modal logics \( K_m \) and \( S_4^m \). We develop a sound and complete tableau algorithm \( \Lambda_K \) for answering \( \mathcal{ALCK}_m \) queries w.r.t. an \( \mathcal{ALCK}_m \) knowledge base with an acyclic TBox. Defining tableau expansion rules in the presence of acyclic definitions by considering only the concept names on the left-hand side of TBox definitions or their negations, allows us to give a PSPACE implementation for \( \Lambda_K \). We then consider answering \( \mathcal{ALCS}_4^m \) queries w.r.t. an \( \mathcal{ALCS}_4^m \) knowledge base (with an acyclic TBox) in which the epistemic operators correspond to those of classical multi-modal logic \( S_4^m \). The expansion rules in the tableau algorithm \( \Lambda_{S_4} \) are designed to syntactically incorporate the epistemic properties. Blocking is incorporated into the tableau expansion rules to ensure termination. We also provide a PSPACE implementation for \( \Lambda_{S_4} \). In light of the fact that the satisfiability problem for \( \mathcal{ALCK}_m \) with general TBox and no epistemic properties (i.e., \( K_{\mathcal{ALC}} \)) is NEXPTIME-complete, we conclude that both \( \mathcal{ALCK}_m \) and \( \mathcal{ALCS}_4^m \) offer computationally manageable and practically useful fragments of \( K_{\mathcal{ALC}} \).

Keywords Description logic · \( \mathcal{ALC} \) · Modal logic · Tableau algorithm · PSPACE