



# Free vibrations of non-uniform and axially functionally graded beams using Haar wavelets

H. Hein\*, L. Feklistova

Institute of Computer Science, University of Tartu, Liivi 2, 50409 Tartu, Estonia

## ARTICLE INFO

### Article history:

Received 9 February 2011

Received in revised form

23 July 2011

Accepted 1 August 2011

Available online 3 September 2011

### Keywords:

Functionally graded material

Non-uniform cross-section

Haar wavelets

## ABSTRACT

Vibrations of non-uniform and functionally graded (FG) beams with various boundary conditions and varying cross-sections are investigated using the Euler–Bernoulli theory and Haar matrices. It is assumed that the cross-section and material properties vary along the beam in the axial direction. The system of the governing equations is transformed with the aid of a set of simplest wavelets. To validate the present results, the non-homogeneity of the beams is discussed in detail and the calculated frequencies are compared with those of the existing literature. The results show that the Haar wavelet approach is capable of calculating frequencies for the beams with different shapes, rigidity, mass density, small or large translational and rotational boundary coefficients. The advantage of the novel approach consists in its simplicity, accuracy and swiftness.

© 2011 Elsevier Ltd. All rights reserved.

## 1. Introduction

In contemporary engineering conditions, the requirements for structural materials and their properties are becoming more stringent. This is particularly true for the materials which are used in constructional elements or assembly units and are utilized in extremely severe environment or adverse exploitation. Nevertheless, traditional means for improving characteristics and performance of natural materials are depleted. Therefore, an increasing interest in composite materials and the materials with gradients in composition is evident. The tendency is also provoked by economical aspects: extraction and processing of natural resources is limited and expensive.

FG materials withstand high temperatures and resist corrosion. On account of comparatively good fracture toughness, FG materials are less exposed to delamination or cracking in comparison to uniform or homogeneous beams; therefore, FG materials have been under important consideration among engineers in recent decades. A detailed overview of the advanced materials, their development, elemental composition, microstructure, properties, design and application are described in [1] by Byrd. A more comprehensive research on thermoelastic behaviour of FG structures was first conducted by Chakraborty et al. in 2003 yet. Static, free vibrations and wave propagation were investigated by the beam element approach which required an exact solution of the static part of the governing differential equations [2]. Aydogdu and Taskin

studied free vibrations of simply supported FG beams with the aid of classical beam theory, parabolic and exponential shear deformation beam theories. The governing equations were found by the Navier type solution [3]. A unified approach for analysing both static and dynamic behaviour of FG beams was proposed by Li extending the Timoshenko beam theory [4]. A fundamental frequency analysis using different higher-order beam theories was carried out by Simsek [5]. In 2010, Alshorbagy et al. suggested FEM for calculating dynamic characteristics of FG beams with material gradation in axially or transversally through the thickness based on the power law [6]. The same method was applied by Shahna et al. for stability analysis of FG tapered Timoshenko beam [7]. Xiang and Yang studied forced vibrations of a three-layer laminated FG Timoshenko beam with arbitrary end supports and varying thickness due to the applied heat [8]. Recently Simsek and Kocatürk studied dynamic behaviour of FG simply-supported beams under a concentrated moving harmonic load. The approach was based on Lagrange's equations [9]. Bending and vibration of cylindrical beams with arbitrary radial non-homogeneity were investigated by Huang and Li [10]. A dynamic system with a moving mass was broadly studied by Simsek in [11,12], and Khalili et al. [13]. A new approach for calculating free vibration of FG beams with non-uniform cross-section area and varying physical properties along its longitude was proposed by Huang and Li [14] last year. The approach was based on the Fredholm integration equation.

The analytical method for studying free vibrations of FG beams was provided by Sina et al. [15] and Mahi et al. [16] only a few years ago. The equation of deflection was derived applying Hamilton's principle. The Galerkin method was employed to analyse free vibration of sandwich beams with FG core in [17].

\* Corresponding author.

E-mail addresses: [helle.hein@ut.ee](mailto:helle.hein@ut.ee) (H. Hein), [nika@ut.ee](mailto:nika@ut.ee) (L. Feklistova).