Engineering Structures 33 (2011) 1850–1854

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Short communication

Geometric properties for the design of unusual member cross-sections in bending

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ARTICLE INFO

Article history:
Received 15 October 2010
Received in revised form
5 January 2011
Accepted 30 January 2011
Available online 9 March 2011

Keywords:
Bending design
Complex cross-sections
Lightweight structures
Second moment of area
Elliptical geometry
Superellipse

ABSTRACT

This short communication presents a series of general geometric properties of unusually-shaped member cross-sections. A skeleton derivation of the most important properties is presented and a method is illustrated for constructing more complex cross-sections. The focus is on the cross-sectional area, centroid and second moment of area, used in classical engineering design for bending. The shapes presented include circular and elliptical segments, sectors and arcs, semiellipses, superellipses, and combinations thereof. The superellipse allows a vast range of novel cross-sections to be described analytically and is a powerful mathematical tool for the analysis of unusual curved shapes. The equations are presented as composites of their geometric components so as to facilitate practising engineers in their implementation as programmed functions or subroutines.

1. Introduction

A series of section properties for non-standard, mono-symmetric, thick-walled, curved hollow beam cross-sections are presented in what follows. The target audience of this work are practising engineers in the light-weight construction, aerospace and sports industries undertaking design and optimisation of unusually-shaped members in bending, usually from cold-formed steel or other ductile material [1]. The shapes presented are circular and elliptical segments, sectors and arcs, semiellipses and superellipses, which when combined allow a very wide range of curved cross-sectional geometries to be analysed. The section properties presented are those required to obtain the radii of gyration and elastic section moduli, used in classical engineering design for bending, notably the cross-sectional area, section centroid and second moment of area [2].

An outline derivation of standard results is presented first, followed by an illustration of how these may be subsequently used as building blocks to construct increasingly complex cross-sectional geometries. Extensive use is made of the parallel axis theorem [2]. The final equations are presented as composite functions of their geometric components, as they would otherwise be very protracted algebraically, in such a way as to facilitate their implementation as programmed functions or subroutines.

2. Standard results for reference shapes

2.1. Circular and elliptical segments

Fig. 1 shows a generalised elliptical segment with major and minor axes a and b respectively, angle $2\theta_G$ such that $\theta_G \leq 1/2\pi$ and centroid located at $(x, y) = (0, y_C)$. Consider an infinitesimally small element of area dA, oriented either vertically or horizontally. The area, centroid and second moment of area about the x and y axes of the full segment may be calculated by $A = \int \mathrm{d}A$, $y_C = \int y x \mathrm{d}y / \int \mathrm{d}A$, $I_x = \int y^2 \mathrm{d}A = \int y^2 x \mathrm{d}y$ and $I_y = \int x^2 y' \mathrm{d}x$ respectively, where $x = a \sin\theta$, $y = b\cos\theta$ and $y' = b(\cos\theta_G - \cos\theta)$. The angle θ is defined from the vertical y axis in the range $-\theta_G \leq \theta \leq \theta_G$. Integrating over the appropriate range of θ yields the following general properties:

$$A_{seg}(a, b, \theta_G) = \frac{1}{2} ab(2\theta_G - \sin 2\theta_G) \quad (1)$$

$$y_{C,seg}(b, \theta_G) = \frac{4b \sin^3 \theta_G}{3(2\theta_G - \sin 2\theta_G)} \quad (2)$$

$$I_{x,seg}(a, b, \theta_G) = \frac{ab^3}{16}(4\theta_G - \sin 4\theta_G) \quad (3)$$

$$I_{y,seg}(a, b, \theta_G) = \frac{a^3 b}{24}(6\theta_G - \sin 2\theta_G(3 - 2\sin^2\theta_G)). \quad (4)$$

These equations simplify to those for a circular segment when $a = b = \rho$, where ρ is the radius. The second moment of area

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0141-0296/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2011.01.026