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Algorithms for seismic analysis of MDOF systems with fractional derivatives

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ABSTRACT

Viscoelastic dampers are often considered for use in structural systems to reduce their dynamic response. The frequency dependent storage and loss moduli of the viscoelastic material are sometimes modeled using the fractional derivatives. This introduces fractional derivatives in the equations of motion. Herein, several schemes, specialized for arbitrary inputs such as earthquake induced ground motions, are presented for direct numerical integration of such equations to obtain the dynamic response of multi-degrees-of-freedom damper–structure systems. Relative numerical accuracies of the proposed schemes are examined. The numerical analyses with fractional derivatives require that all previous time response values be used, but this is invariably time consuming. The possibility of discarding some early time response values without compromising the accuracy of the calculations is, thus, of natural interest. It is shown that such truncations are, indeed, possible but a straightforward omission of preceding time values can introduce unacceptable errors in the calculated responses. To reduce such errors a new algorithm is proposed.

1. Introduction

Passive energy dissipation is, now, a well-accepted approach for response reduction of vibration systems. The energy dissipation in structural systems is realized through the use of suitably placed dampers. Currently, several different types of dampers are being considered for application in structures. The dampers that use viscoelastic materials are among the popular types of dampers and they have been used to retrofit deficient buildings as well as installed in newly designed buildings. This paper is concerned with the analysis of structural or mechanical systems that utilize the viscoelastic dampers to reduce their vibratory responses caused by earthquake induced ground motions.

It is now recognized that the stiffness and energy dissipation characteristics of viscoelastic dampers that are of interest in dynamic analysis are frequency and temperature dependent. To represent the frequency dependence of these properties, models with varying degrees of complexity have been used. A model that has been used to accurately capture the frequency dependence of the properties is the fractional derivative model [1]. Bagley and Torvik [2] and Torvik and Bagley [3] suggested the use of the following three-parameter Kelvin model with fractional derivatives to represent the force–deformation relationship of the viscoelastic dampers to include the frequency dependence of the material properties:

$$f_{\mathrm{ve}} = \bar{k}u_{\mathrm{ve}} + \bar{c}D^{\alpha}\langle u_{\mathrm{ve}}\rangle, \quad 0 < \alpha < 1 \tag{1}$$

where $\bar{k}$ and $\bar{c}$ are the stiffness and damping coefficients of the damper, the parameter α is the fractional derivative order of the viscoelastic material, and $D^{\alpha}\langle\cdot\rangle = \mathrm{d}^{\alpha}/\mathrm{d}t^{\alpha}$ denotes the fractional derivative operator.

There are two basic definitions for the fractional derivative. One definition, offered by Grunwald [4], is of the following form:

$$D^{\alpha}\langle f(t)\rangle = \lim_{N\to\infty}\left\{\left(\frac{t}{N}\right)^{-\alpha}\sum_{j=0}^{N-1}\frac{\Gamma(j-\alpha)}{\Gamma(-\alpha)\Gamma(j+1)}f(t-jt/N)\right\}. \tag{2}$$

The other definition is in terms of the following Riemann–Liouville integral [4]:

$$D^{\alpha}\langle f(t)\rangle = \frac{1}{\Gamma(1-\alpha)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha}}\mathrm{d}\tau \tag{3}$$

where $\Gamma(.)$ is the gamma function. It can be shown that these two definitions are equivalent [5].

For a multi-degree-of-freedom structure installed with viscoelastic dampers modeled in terms of fractional derivatives, the equations of motion under seismic excitation can be written in the following general form:

$$\mathbf{M}\{\ddot{x}\} + \mathbf{C}\{\dot{x}\} + \bar{\mathbf{C}}\{D^{\alpha}\langle x\rangle\} + \mathbf{K}_t\{x\} = -\mathbf{M}\{r\}\ddot{x}_g(t) \tag{4}$$

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