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frequencies of the bridge as a function of the amplitude of vibration.

# A simple finite element to consider the non-linear influence of the ballast on vibrations of railway bridges

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ABSTRACT

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## 1. Introduction

In the dynamic behavior of railway bridges, discrepancies between calculations and experiments are often observed. This problem especially applies to short or median span bridges for which important differences between calculated and measured natural frequencies can be obtained. Several studies, see e.g. [1,2], have also shown that, for such bridges, the natural frequencies vary as function of the amplitude of vibration.

One difficulty in modeling relatively short railway bridges is that the influence of the track superstructure composed by rails, sleepers and ballast is not well known. For example, there is, so far, no clear recommendation in design codes to take the ballast into account in dynamic analyses.

In several works about train-track-bridge dynamic interaction [3–6], the track and the bridge have been modeled by two beams and the effect of the ballast has been introduced using a more or less advanced system of viscoelastic spring/dampers and masses between the two beams. In [7], Liu et al. developed a 3D finite element model of the bridge using elastic solid elements for the ballast.

Müller et al. [8] and Ruge and Birk [9], Ruge et al. [10] developed a truss model to study the longitudinal stresses due to temperature changes and train braking. Following the European codes, they introduced a non-linear stiffness between the bridge and the rail to represent the coupling effect of the ballast.

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0141-0296/\$ – see front matter © 2011 Elsevier Ltd. All rights reserved. doi:10.1016/j.engstruct.2011.05.005 The purpose of the present article is to propose a new and simple approach to model the effect of the ballast in vertical dynamic analyses of railway bridges. The idea is to use the methodology in [8–10] to develop a simple 2D beam finite element. In this element, the effect of the ballast is taken into account by introducing a linear or non-linear longitudinal stiffness associated to the slip at the interface between the bridge and the ballast. This approach has already been presented by Fink and Mähr [11], but in a continuum context, using differential equations.

The organization of the paper is as follows: the derivation of the finite element is presented in Section 2. In Sections 3 and 4, two numerical applications, based on lab and in-situ experiments are proposed. In particular, it is shown that the present element can be used to model the variation of the lowest bending natural frequency as a function of the amplitude of vibration. Finally, conclusions are presented in Section 5.

## 2. Finite element formulation

This article proposes a new and simple finite element which can be used to analyze vertical vibrations in

railway bridges. The main feature of the element is that the effect of the ballast is introduced through a

non-linear longitudinal stiffness associated to the slip at the interface between the bridge and the ballast.

Two numerical applications show that this element can be used to model the variation of the natural

The finite element, see Figs. 1 and 2, consists of two layers. The layer (a) represents the bridge whereas the layer (b) represents the track superstructure composed by rails, sleepers and ballast. The slip at the interface between the two layers is considered. The element has four degrees of freedom. The horizontal displacements of the neutral axes of the two layers are not considered. It is assumed that the vertical displacements v and rotations  $\theta$  of the neutral axis of the two layers are the same. The Bernoulli hypothesis is adopted, together with cubic shape functions, which





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