#### Engineering Structures 33 (2011) 3238-3245

Contents lists available at SciVerse ScienceDirect



**Engineering Structures** 



journal homepage: www.elsevier.com/locate/engstruct

# Shear and bending flexibility in closed-form moment solutions for continuous beams and bridge structures

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#### ARTICLE INFO

Article history: Received 27 July 2011 Received in revised form 9 August 2011 Accepted 10 August 2011 Available online 23 September 2011

Keywords: Continuous beam Frame Bridge Moment Shear deformation Statically indeterminate Moment distribution Stiffness method Exact

#### ABSTRACT

Shear deformations can affect final member-end-moments for statically indeterminate continuous beams and frame structures, though for typical civil engineering structures their effect is small and moments can be based on flexural deformations only. When a member is deep relative to span length, however, shear deformations should be considered in the analysis. This can be included in the stiffness method and in a modified form of moment distribution where the carry-over factor is less than one-half due to the added flexibility from shear. In a prior paper the first author presented a new approach for solving statically indeterminate beams and bridge frames, with final end moments given in closed-form expressions. The advantages of this new approach are that no simultaneous equations are required as in the stiffness method, moments are not distributed back and forth as in moment distribution, and manual calculations were considered in the original paper, this paper presents a closed-form approach that has been modified to include shear deformations. Final expressions are given for continuous beams and bridge frames, providing exact member-end-moments that match results from the stiffness method when shear deformations are included in the analysis.

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### 1. Introduction

Typical continuous beams and frames of civil engineering structures are designed with members that are long relative to their depth, causing flexural deformations to be of much more significance than shear deformations. Thus, it is often not necessary to consider shear deformations when solving for member forces of a statically indeterminate structure and instead base results completely on the bending behavior of the members. This is true in the flexibility method, stiffness method and in moment distribution. Of these methods, moment distribution is the simplest approach for manual calculations as no simultaneous equations are involved [1-3]. With final memberend-moments known, shear and axial forces may subsequently be found from statics, allowing moment and shear diagrams to be developed. For typical dimensions (small depth-to-span ratio), realistic shear forces are determined regardless of whether or not shear deformations are included in the analysis.

When members are deep in relation to their length, shear flexibility has an impact on the final distribution of member-endmoments for statically indeterminate beams and frames, changing the moment and shear diagrams for all members and axial forces of frame members. For determinate structures member depth has no influence on moment, shear and axial force diagrams. In a prior paper by the first author a closed-form approach was presented that gave final member-end-moments for continuous beams and bridge structures (Fig. 1) in simple expressions that can be used to calculate exact results manually [4].

The assumption in the original derivation was that shear deformations are small compared to flexural deformations and can be neglected, which is reasonable for many structures. In this paper, however, the closed-form equations presented in [4] have been modified to include shear deformations, giving identical results as the stiffness method for deep sections. Closed-form expressions presented herein can also be used for non-prismatic members, although this is not the subject of the present paper and would require variations to standard fixed-end-moments, rotational stiffness expressions and carry-over factor terms.

In the original paper [4] it was shown that the closed-form approach allows exact solutions to problems that could not otherwise be determined, requiring the solution of an infinite number of simultaneous equations in the stiffness method or an infinite number of cycles in moment distribution. For example, imagine a continuous beam with an infinite number of equallength spans with one span loaded. Simple equations for final

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