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Least-work solutions of flange normal stresses in thin-walled flexural members with high-order polynomial

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ABSTRACT

An energy-based method was developed for quantifying shear lag effects in thin-walled flexural members such as box girders, T-beams, and nonrectangular concrete walls. The proposed procedure uses infinite terms of high-order polynomial to describe the uneven longitudinal displacement in the flanges. The series type of approximation resulted in a group of coupled differential equations, for which solution techniques were developed. The proposed variational analysis was compared with the existing least-work solutions and two experimental tests of rectangular box girders in the literature and one of tests of steel box beams in this study. The comparisons indicated that the proposed variational analysis can accurately predict the flange normal stresses in box girders. Solutions were provided for thin-walled flexural members in bridges and buildings under a variety of loadings and boundary conditions to facilitate the implementation of the proposed procedure.

1. Introduction

The shear lag effect in this paper describes the unevenly distributed normal stress in the flange of a thin-walled flexural member, as shown in Fig. 1. Due to shear lag effects, the structural behavior can be different from that predicted by the elementary beam theory, which assumes that the normal stress in a slender beam is proportional to the distance from the neutral axis. Shear lag has been identified and studied in many engineering structures, including airplane structures [1–3], high-rise buildings [4–6], composite beams [7–12], and box girder bridges [13–15]. Two concepts, effective flange width [10–12,16,17] and stress increase factor [9,18,19], are widely used in engineering design practices. Effective flange width is the partial flange in tension/compression, with which the largest normal stress in a thin-walled flexural member can be obtained following the elementary beam theory. The stress increase factor modifies the normal stress calculated using the original cross section and the elementary beam theory such that the peak normal stress in the flange can be obtained for design. The determination of both design parameters requires an accurate normal stress distribution across the flanges of thin-walled flexural members.

The existing shear lag analysis techniques include analytical procedures (e.g., the finite stringer method [20,21], the biharmonic analysis [1,9,22], and energy-based analyses [2,6,14,18,23,24]), numerical analyses [10–13,17,19], and experimental tests [18,25]. The results of energy-based analyses have been used in the design of box girder bridges and high-rise buildings [5,6,14,26]. In the existing energy-based analysis, the longitudinal flange displacement is described using a quadratic or cubic polynomial term with one unknown parameter. The variational principle is applied to the potential energy of the member to determine the unknown parameter. The accuracy of the energy-based analyses is thus limited by the ability of the assumed polynomial to approach the longitudinal displacement across the flanges, which varies along the member as observed in several studies [19,27].

A $2m$-degree polynomial (the summation of m terms of binomials with even exponents) was used in this study to approach the actual longitudinal displacement in the flanges of thin-walled flexural members with various boundary conditions and loadings. The proposed series type of approximation resulted in a group of coupled differential equations, for which solution techniques were developed by solving an eigenvalue problem. This procedure is described below following a review of the existing analysis techniques. The analyses of two steel box girders tested and documented in the literature and a steel box beam in this study were used to demonstrate the effectiveness of the proposed procedures.

2. Literature review

Analytical procedures are needed to provide guidance for experimental tests and numerical studies. Among the existing

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