Engineering Structures 33 (2011) 298–307

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Derivation of the exact stiffness matrix for a two-layer Timoshenko beam element with partial interaction

Quang-Huy Nguyen [a], Enzo Martinelli [b], Mohammed Hjiaj [a,*]

[a] Structural Engineering Research Group, INSA de Rennes, 20 avenue des Buttes de Coësmes, CS 70839, F-35708 Rennes Cedex 7, France

[b] Department of Civil Engineering, University of Salerno, Via Ponte don Melillo - 84084 Fisciano (SA), Italy

ARTICLE INFO

Article history:
Received 31 March 2010
Received in revised form
23 August 2010
Accepted 4 October 2010
Available online 18 November 2010

Keywords:
Two-layer beam
Steel–concrete composite beam
Partial interaction
Shear deformation
Closed-form solution
Exact stiffness matrix

ABSTRACT

This paper presents the full closed-form solution of the governing equations describing the behaviour of a shear-deformable two-layer beam with partial interaction. Timoshenko's kinematic assumptions are considered for both layers, and the shear connection is modelled through a continuous relationship between the interface shear flow and the corresponding slip. The limiting cases of perfect bond and no bond are also considered. The effect of possible transversal separation of the two members has been neglected. With the above assumptions, the present work can be considered as a significant development beyond that available from Newmark et al.'s paper [4]. The differential equations derived considering the above key assumptions have been solved in closed form, and the corresponding "exact" stiffness matrix has been derived using the standard procedure basically inspired by the well-known direct stiffness method. This "exact" stiffness matrix has been implemented in a general displacement-based finite element code, and has been used to investigate the behaviour of shear-deformable composite beams. Both a simply supported and a continuous beam are considered in order to validate the proposed model, at least within the linear range. A parametric analysis has been carried out to study the influence of both shear flexibility and partial interaction on the global behaviour of composite beams. It has been found that the effect of shear flexibility on the deflection is generally more important for composite beams characterized by substantial shear interaction.

1. Introduction

For the last few decades, composite members and structures have often been used in civil engineering. Steel–concrete composite beams and nailed timber members are two possible technical solutions based on coupling two layers made up of different materials with the aim of optimizing their mechanical behaviour within a unique member. For these applications, relative displacements generally occur at the interface of the two layers, resulting in the so-called partial interaction. Whereas the transverse separation is often small and can be neglected [1,2], interface slips influence the behaviour of two-layer composite beams and must be considered. Several theoretical models characterized by different levels of approximation have been proposed and are currently available within the scientific literature. Timoshenko [3] developed a theory for composite beams with two bonded materials using the Bernoulli–Euler beam model for each component and assuming no relative displacement between them. The first formulation of an elastic theory for composite beams with partial interaction is commonly attributed to Newmark et al. [4]. They adopted the Euler–Bernoulli kinematic assumptions for both the concrete slab and the steel profile, and considered a continuous and linear relationship between the relative interface displacements (slips) and the corresponding interface shear stresses. This formulation is usually referred to as Newmark's model, which is the most cited work in the area of composite beams with continuous shear connection. In their paper, a closed-form solution is provided for elastic composite beams. Since then, this model has been used extensively by many authors to formulate analytical models for the static response of the composite beams in the linear-elastic range [5–11] as well as in the linear-viscoelastic range [12–16]. Buckling loads for composite members have been derived by Möhler [17]. Newmark's model was further developed to deal with the dynamic response of composite beams, which in some situations governs the design [18–20]. Besides these analytical works, several numerical models based on the basic assumptions of Newmark's model have been developed to investigate the behaviour of composite beams with partial interaction in the nonlinear range (for material nonlinear models, see, e.g., [21–25]; for geometric nonlinear models, see, e.g., [26–29]). Most of these papers are concerned with finite element (FE) formulations. A closed-form solution leading to an "exact" finite element, formally conceived under the so-called

* Corresponding author.
E-mail address: mohammed.hjiaj@insa-rennes.fr (M. Hjiaj).

0141-0296/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2010.10.006