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# Nonlinear dynamic behavior of saddle-form cable nets under uniform harmonic load

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#### 1. Introduction

Cable nets belong to the family of tensile structures that are characterized by their capacity to carry loads much heavier than their own weight. The most common shape of cable nets is the hyperbolic paraboloid. The net consists of two families of cables: the carrying or main cables that produce the concave surface, which are anchored at the highest points of the boundary, and the stabilizing or secondary ones, which, anchored at the lowest points of the boundary, create the convex surface. Cable structures differ from conventional linear systems, due to their nonlinear response to both static and dynamic actions. Their response cannot be obtained on the basis of their original undeformed geometry, because their stiffness increases as the deflection increases, and the internal forces do not vary linearly with load. Therefore, it is necessary to take into consideration the deformed state at every step of the load, performing nonlinear analyses, which account for large displacements.

The dynamic response of a nonlinear system is unpredictable, as several nonlinear phenomena may appear, such as secondary resonances, including superharmonic and subharmonic resonances,

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## ABSTRACT

The dynamic response of saddle-form cable nets is investigated in this paper. Even though they consist of cables, which are well known for their geometric nonlinearity, such systems could be characterized as weakly nonlinear due to the high levels of pretensioning of their cables and to their hyperbolic paraboloid surface, having opposite curvatures at all points and thus increased stiffness. Nevertheless, resonance phenomena that are typical of highly nonlinear systems are detected here, for common geometries and levels of pretension, even for low levels of load amplitude. First, a single-degree-of-freedom (SDOF) cable net is studied analytically and numerically, and nonlinear resonances are confirmed. Then, the response of multi-degree-of-freedom (MDOF) cable nets, subjected to harmonic dynamic excitation, is investigated. Although the static response is proved to be almost linear, the dynamic nonlinearity is intense, as verified by jump phenomena, bending of the response curve, superharmonic resonances, and dependence on the initial conditions.

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depending on the relation between the loading frequency  $\Omega$  and the eigenfrequencies  $\omega_i$  of the system. If  $\Omega \approx n \cdot \omega_i$  or  $\Omega \approx (1/n) \cdot$  $\omega_i$ , where *n* is an integer, subharmonic or superharmonic resonance may occur, respectively. In such cases, all modes involved in the secondary resonances are activated during the oscillation [1,2]. The relation between oscillation amplitude and frequency can be described by a response diagram, in which the steady-state amplitude is plotted on the vertical axis and the frequency ratio  $\Omega/\omega$  on the horizontal axis, where  $\Omega$  is the loading frequency and  $\omega$  the natural frequency of the system. For a free vibration of an undamped oscillator, the steady-state response is described by one line, known as the backbone. For a forced system, the steady-state response is represented by different curves, depending on the amplitude of the external force. These curves can be interpreted as perturbations out of the equilibrium state. In linear systems, the backbone is a straight vertical line and the response curves for the forced systems approach this line asymptotically, as the forcing frequency  $\Omega$  approaches the system's frequency  $\omega$ , indicating the phenomenon of fundamental resonance, in which the vibration amplitude increases infinitely when the force has the same frequency as the system. In nonlinear systems instead, the backbone is a bending curve accounting for either the softening or the hardening behavior of the system (Fig. 1). The softening behavior means that the stiffness of the system decreases as the oscillation amplitude increases, while in the hardening behavior the system's stiffness becomes progressively higher for large amplitudes [3].

Several studies have been published in the past regarding nonlinear dynamic phenomena for individual cables [4–6], while some





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