THE EFFECT OF ROUNDED CORNERS ON THE SECONDARY FLOWS OF VISCOELASTIC FLUID IN NON-CIRCULAR DUCT

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Abstract

In present work, the Reiner-Rivlin viscoelastic model is used to investigate numerically the pattern and strength of the secondary flows in rounded square duct as well as the influence of rheological properties on them. The governing equations for steady state, laminar fully developed flow have been solved by finite difference method. It is shown the secondary flow of a Reiner-Rivlin fluid has a negligible effect on the axial flow and *fRe.* Viscous and elastic behavior influence the magnitude of the secondary flow. The results indicate that the rounding of the corners has important effect on the secondary flow.

Keywords: Secondary flow, Viscoelastic, Reiner-Rivlin model, Rounded square duct

Introduction

Understanding of viscoelastic fluid flow becomes increasingly important due to wide range of application of these fluids in many industries such as polymer processing. In flow of viscoelastic fluid through noncircular channels under laminar conditions, it has been predicted that secondary flows would occur reflecting the fact that the stresses acting on orthogonal faces of a fluid element are not equal [1]. The existence of such secondary flows has also demonstrated by means of numerical simulation using the Criminale-Ericksen-Fibley (CEF) model [2, 3, 4, 5], the Reiner-Rivlin model [6] and Phan-Thien-Tanner (PTT) model [7].

Dodsen et al. [8] have used CEF fluid model and have solved governing equations by perturbation method where the solution expanded in powers of second normal stress coefficient ψ_2, and they reported the nature of the secondary flows for small values of ψ_2. They reported also the effect of the secondary flows on pressure drop is small at low flow rates and but more significant at high flow rate. There are in total two vortices in each quadrant of the rectangular duct at different aspect ratio, and at each ratio the pattern of secondary flows takes the same form for different material parameters, but their strength is very sensitive to the viscose and elastic effects of viscoelastic fluid. Increasing aspect ratio causes four of eight vortices become considerablely weaker.

With this background the effect of rectangular duct aspect ratio on secondary flow of viscoelastic fluid has been reported by several researchers. Also their results demonstrate that elasticity plays no role in the case of the circular tube geometry. But the effect of sharp corner on pattern and strength of secondary flow of viscoelastic fluid has not yet been reported. The present study is to investigate numerically the laminar flow of Reiner-Rivlin viscoelastic through square duct with different roundness of corners.

Formulation

Figure 1 shows a schematic diagram of the system under consideration. Constant property, fully developed and laminar flow of Reiner-Rivlin viscoelastic fluid model are considered. Thus, the governing equation can be expressed [9]:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equations:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{xx}}{\partial x} \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y} \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}\right)=-\frac{\partial p}{\partial z}+\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{zy}}{\partial y} \tag{4}$$

Stress tensor component is obtained from Reiner-Rivlin viscoelastic fluid model.

$$\tau=-\eta\dot{\gamma}-\psi_2\dot{\gamma}\cdot\dot{\gamma} \tag{5}$$

Where $\dot{\gamma}$ is shear rate tensor:

$$\dot{\gamma}=\nabla v+\nabla v^{\dagger} \tag{6}$$

Where ∇v is velocity gradient tensor and η is the apparent viscosity for generalized non-Newtonian model can be calculated from power law viscosity model:

$$\eta=K\dot{\gamma}^{\,n-1} \tag{7}$$