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Unsteady flows of inhomogeneous incompressible fluids

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ABSTRACT

In this paper, we study the unsteady motion of inhomogeneous incompressible viscous fluids. We present the results corresponding to Stokes' second problem and for the flow between two parallel plates where one is oscillating.

1. Introduction

One of the main areas of interest in fossil energy related applications, such as power plants, atomization, biomass, alternative fuels, etc., is the use of slurries, specifically coal–water or coal–oil slurries, as the primary fuel. Some studies indicate that the viscosity of coal–water mixtures depends not only on the volume fraction of solids, the mean size, and the size distribution of the coal, but also on the shear rate, since the slurry behaves as a non-Newtonian fluid (see Massoudi and Phuoc [1]). At the same time, there are studies which indicate that preheating the fuel results in better performance (see Gupta and Massoudi [2]) and as a result of such heating, the viscosity changes. The non-linear time-dependent response of complex fluids (such as drilling fluids, coal–water slurries, electro-rheological fluids, and polymers) constitutes an important area of mathematical modeling of non-linear fluids.

Exact (or analytical) solutions to the steady or unsteady flow of a fluid play a significant role not only in understanding the behavior and the response of the fluid, but also in helping the design of experiments to measure material properties. Furthermore, exact solutions establish benchmark examples to test the accuracy and validity of the numerical schemes used in studying the more sophisticated models or more complicated geometries. Most of the exact solutions to the full Navier–Stokes equations and to the simplified boundary layer equations are given in classical textbooks on fluid mechanics, such as Batchelor [3], White [4], and Schlichting [5], and in review articles by Boulanger et al. [6], and Wang [7]. For many non-linear fluids such exact solutions either do not exist, depending on the model, or are rare and not as easily obtainable [8–12].

Rajagopal [11] obtained exact solutions for a class of unsteady unidirectional flows where the fluid was assumed to behave as an incompressible second grade fluid. These solutions correspond to (a) the flow due to an oscillating rigid plate in its own direction (also known as Stokes' second problem), (b) the flow between two parallel plates one of which is oscillating, (c) the flow between two rigid walls one of which is suddenly set into motion, and (d) the time periodic Poiseuille flow where there is an oscillating pressure gradient.

In this short paper, we study the unsteady motion of inhomogeneous incompressible viscous fluids, where the viscosity varies spatially. We present the results corresponding to Stokes' second problem and for the flow between two parallel plates where one is oscillating.

2. Governing equations

In the absence of thermo-chemical and electro-magnetic effects, the governing equations of motion are the conservation of mass and linear momentum equations. These are (see Truesdell and Noll [13])

Conservation of mass:

$$\frac{\partial \rho}{\partial t} + div(\rho \mathbf{u}) = 0 \quad (1)$$

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