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Unsteady flow in a circular sector duct

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ABSTRACT

Analytic solutions for the unsteady flow in a circular sector duct are found using series sums of Bessel integrals. For starting flow due to a step pressure gradient, the velocity profile is initially flat, then approaches the rounded steady state shape in a time scale proportional to the square of opening angle of the sector. For oscillatory flow, the velocity is quasi-steady for low frequencies, but shows "annular effect" at large frequencies. Increased opening angle increases the amplitude and the phase lag. In all cases, the shear stress at the apex is zero for acute sector angles but becomes infinite for obtuse sector angles.

1. Introduction

Unsteady, analytic solutions to the Navier–Stokes equations are very few. See a review by Wang [1]. The unsteady parallel flow equation, although simpler than the Navier–Stokes, still present strong challenges. These flows may be classified into two types. The first type describes the flow due to unsteady parallel motion of a boundary, called the Rayleigh problem or Stokes' problem. The solutions are also related to the unsteady heat conduction on a solid boundary. Previous literature include tangential motion of parallel plates and circular cylinders [2], the circular sector [3], the wedge [4,5] and the rectangle [6]. The second type is the flow in a duct due to an unsteady pressure gradient. The starting flow in a circular tube was solved by Szymanski [7]. The solution is also related to that of the sudden internal heat generation in solids [2]. Other cross sections such as rectangle or semi-circle was solved by Erdogan [6], Erdogan and Imrak [8]. The flow due to an oscillatory pressure gradient in a circular pipe was solved by Grace [9] and Sexl [10], and the annular duct by Tsangaris [11]. Other cross sections include the elliptic tube by Khamrui [12] and Haslam and Zamir [13], the rectangular duct by O'Brien [14] and the equilateral right triangle by Tsangaris and Vlachakis [15].

In this paper, we consider the analytic solution of the flow due to unsteady pressure gradient in a circular sector duct. The sector duct includes the semi-circle and the circle with a radial fin, and is most suitable for the study of the unsteady flow properties near acute or obtuse angles. The general solution was given by Hepworth and Rice [16], in terms of double series and Duhamel integrals. However, only some specific profiles of the starting flow in a semi-circular duct are presented. In this paper, both starting flow and oscillatory flow for different sector angles will be compared.

2. The starting flow

We generalize the method of Erdogan and Imrak [8] for the semi-circle to the sector duct. Fig. 1 shows the cross section of the duct, with radius L and opening angle 2β. For parallel flow, the Navier–Stokes equation reduces to

$$w'_{t'} = G'/\rho + \nu\nabla^2 w' \quad (1)$$

where w' is the longitudinal velocity, t' is the time, G' is the negative pressure gradient, ρ is the density, and ν is the kinematic viscosity. The boundary conditions are that the velocity is zero on the walls. For starting flow, the pressure gradient is a step function

$$G' = GH(t) \quad (2)$$

where G is a constant and $H(t)$ is a unit step. Normalize all lengths by L, and the time by L^2/ν, the velocity by $GL^2/(\rho\nu)$ and drop primes. Eq. (1) becomes

$$w_t = H(t) + w_{rr} + w_r/r + w_{\theta\theta}/r^2 \quad (3)$$

The steady flow equation is

$$0 = 1 + \overline{w}_{rr} + \overline{w}_r/r + \overline{w}_{\theta\theta}/r^2 \quad (4)$$

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