Contents lists available at ScienceDirect



International Journal of Non-Linear Mechanics



journal homepage: www.elsevier.com/locate/nlm

# Unsteady stagnation-point flow of a Newtonian fluid in the presence of a magnetic field

# F. Labropulu\*

Luther College—Mathematics, University of Regina, Regina, SK, Canada S4S 0A2

#### ARTICLE INFO

## ABSTRACT

Article history: Received 8 December 2008 Accepted 5 April 2011 Available online 15 April 2011

Keywords: Unsteady Newtonian Magnetic field Oscillations The unsteady stagnation-point flow of a viscous fluid impinging on an infinite plate in the presence of a transverse magnetic field is examined and solutions are obtained. It is assumed that the infinite plate at y=0 is making harmonic oscillations in its own plane. A finite difference technique is employed and solutions for small and large frequencies of the oscillations are obtained for various values of the Hartmann's number.

© 2011 Elsevier Ltd. All rights reserved.

### 1. Introduction

The flow of an incompressible viscous fluid over a moving plate has its importance in many industrial applications. The extrusion of plastic sheets, fabrication of adhesive tapes and application of coating layers onto rigid substrates are some of the examples. If a magnetic field is present, viscous flows due to a moving plate in an electro-magnetic field, i.e. magnetohydrodymanic (MHD) flows, are relevant to many engineering applications such as petroleum engineering, chemical engineering, MHD power generators, MHD pumps, heat exchangers and metallurgy industry. Examples of MHD flows in the metallurgy industry include the cooling of continuous strips and filaments drawn through a quiescent fluid and the purification of molten metals from non-metallic inclusions.

In the history of fluid dynamics, considerable attention has been given to the study of two-dimensional stagnation-point flow. Hiemenz [1] derived an exact solution of the steady flow of a Newtonian fluid impinging orthogonally on an infinite plate. Stuart [2], Tamada [3] and Dorrepaal [4] independently investigated the solutions of a stagnation-point flow when the fluid impinges on the plate obliquely. The flow over a moving infinite plate is essentially a two-dimensional stagnation-point flow. This consists of a class of flows in the vicinity of a stagnation line that results from a two-dimensional flow impinging on a surface at right angles and flowing thereafter symmetrically about the stagnation-point. Furthermore, the unsteady or time dependent

E-mail address: fotini.labropulu@uregina.ca

0020-7462/\$ - see front matter  $\circledcirc$  2011 Elsevier Ltd. All rights reserved. doi:10.1016/j.ijnonlinmec.2011.04.006

viscous flow near a stagnation-point has also been widely investigated. Glauert [5] and Rott [6] studied the stagnation-point flow of a Newtonian fluid when the plate performs harmonic oscillations in its own plane. The Hiemenz flow of a Newtonian fluid in the presence of a magnetic field was first considered by Na [7] and later on by Ariel [8].

In this paper, we consider the unsteady two-dimensional flow of a viscous incompressible fluid impinging on an infinite plate in the presence of a magnetic field. The plate is assumed to make harmonic oscillations in its own plane. The magnetic field is assumed to be transverse or perpendicular everywhere in the flow field. With the use of a transformation, the governing equations are reduced to a system of ordinary differential equations. Solutions for small frequencies of the oscillations are obtained using a finite difference technique. Solutions for large frequencies are found using a series expansion.

#### 2. Flow equations

The two-dimensional flow of a viscous incompressible fluid in the presence of a magnetic field is governed by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial x} = v \nabla^2 u - \frac{\sigma B_0}{\rho} u$$
(2)

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial y} = v \nabla^2 v$$
(3)

<sup>\*</sup> Tel.: +1 306 585 5040; fax: +1 306 585 5267.