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Constitutive structure in coupled non-linear electro-elasticity: Invariant descriptions and constitutive restrictions

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ABSTRACT

A constitutive framework for electro-sensitive materials in the context of non-linear elasticity is analyzed. Constitutive equations are given in terms of energy functions that depend on several invariants. The study includes both the analysis of the invariants, which are present in the energy functions, and the analysis of constitutive restrictions that have to be obeyed by the constitutive functions. Isotropic as well as non-isotropic electro-sensitive elastomers are studied. The set of invariants that describe each material model is analyzed under two homogeneous deformations: (i) an uniaxial elongation and (ii) a simple shear deformation. These deformations are chosen since they are relevant to specific experiments, from which one may try to fit constitutive equations. The constitutive restrictions developed are based on classical ones used for isotropic non-linear elastic materials, in particular, are based on the Baker–Ericksen inequality and the ellipticity condition.

1. Introduction

In recent years there has been an increasing interest in coupled non-linear electro-elasticity due, in particular, to some new electro-sensitive elastomers, that have recently been developed [1,2]. These elastomers have been applied to the design of actuators and in robotics [3–5]. In addition, a lot of attention has been paid to the development of theoretical frameworks, able to capture strong coupling between deformations and electric fields inside not only these elastomers, but also in the analysis of other materials such as the human heart tissue [6].

One way to produce electro-active elastomers is to mix a highly elastic material, such as rubber, with electro-active particles. The application of an external electrical field generates electrical forces in the electro-active particles that cause the deformation of the body [1]. In some biological constitutive equations, for instance heart wall tissue, the structure of these models has considered a matrix material together with several preferred fiber reinforcement directions [7]. Although the origins are not well understood, parts of the heart wall tissue react to the presence of electric fields causing contraction or dilatation. It is a complex phenomenon, where we can distinguish (among other factors) a possible coupling between deformation and electric fields [6]. Here and thereafter we will consider a matrix (or host material) filled with either particles or fibers, where matrix and particles (or fibers) may or may not be electrically active.

The mathematical modeling of non-linear electro-active materials under large deformations is a difficult task. Different theories have been recently proposed [8–17]. In this work we further exploit the formulation developed by Dorfmann and Ogden [12,13], in which the existence of an amended total energy function is assumed in a parallel formulation to the classical non-linear elasticity framework.

We consider a total energy function that depends on the deformation gradient and one of the electric variables (either the electric field or the electric displacement). If electro-active particles are randomly distributed inside the matrix, it can be shown that the energy function depends on six invariants [12,13]. On the other hand, if the matrix is filled with particles that are aligned in a preferred direction, it can be shown that the energy function depends on 10 invariants [10,18]. More complicated cases can be considered. One may have a matrix filled with two families of fibers, where it may or may not be assumed that matrix and/or fibers react to electric fields. In all these cases we end up with an energy function that can depend on a large number of invariants, which makes particularly difficult to propose and to fit specific forms of the constitutive equations using possible available experimental data. Furthermore, it is justifiable to question the efficacy of a model whose material model depends on many invariants. The highly non-linear nature of the equations involved make numerical methods an essential tool to model the behavior of these materials (see, for example, [19]). Suffice to say that it is merely impossible to

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