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Pressure-driven flow of a rate-type fluid with stress threshold in an infinite channel

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ABSTRACT

In this paper we extend some of our previous works on continua with stress threshold. In particular here we propose a mathematical model for a continuum which behaves as a non-linear upper convected Maxwell fluid if the stress is above a certain threshold and as a Oldroyd-B type fluid if the stress is below such a threshold. We derive the constitutive equations for each phase exploiting the theory of natural configurations (introduced by Rajagopal and co-workers) and the criterion of the maximization of the rate of dissipation. We state the mathematical problem for a one-dimensional flow driven by a constant pressure gradient and study two peculiar cases in which the velocity of the inner part of the fluid is spatially homogeneous.

1. Introduction

A large variety of materials such as food products, polymers, paints, oils and foams cannot be described by the classical linear viscous model. For this reason a large class of non-Newtonian models have been developed with the aim of explaining non-linear behaviors such as shear thinning/thickening, non-linear creep and stress relaxation.

In his celebrated work [7] Maxwell developed the first rate-type fluid model capable of describing stress relaxation, while later Burgers [1] developed a rate-type model for describing some geomaterials which included the classical rate-type model due to Oldroyd [8], namely the Oldroyd-B type model. Oldroyd was the first to develop a consistent framework for the rheology of rate-type viscoelastic fluids, focussing on the importance of the frame invariance and introducing some kinds of derivatives to obtain proper frame indifferent constitutive equations.

Since these seminal works, a plethora of models for viscoelastic response have been developed and numerous frame-invariant time derivatives have been introduced. Rajagopal and co-workers have developed in [10] a proper thermodynamical framework from which most of the viscoelastic constitutive relations can be derived.

The laminar flow of rate-type fluids have been extensively studied both in planar and cylindrical geometries. Waters and King [16] studied the pressure driven flow of an Oldroyd-B fluid in a straight cylindrical pipe, obtaining exact solutions by means of Laplace transform method. Rahaman and Ramkissoon [9] have studied the non-stationary flow of a Maxwell fluid in a pipe. Steady solutions due to oscillating cylindrical boundaries for second grade and Oldroyd-B type fluids have been obtained by Rajagopal [12] and Rajagopal and Bhatnagar [13].

In this paper we investigate the behavior of a non-Newtonian incompressible rate-type fluid which switches from an Oldroyd-B behavior to a non-linear Maxwell behavior depending on whether the stress is larger or smaller than a certain threshold. A typical example of a continuum that changes its behavior depending on the value of some function of the stress is the so-called Bingham fluid, which is a Newtonian viscous fluid that exhibits a threshold (the so-called yield stress) below which the strain rate is zero (so that no deformations occur).

In previous works we have studied a series of extensions of this simple model and we have investigated the corresponding mathematical problems in one-dimensional settings. The first extension was to the case in which the region where the stress is below the threshold behaves like a Neo-Hookean elastic solid (see [2]) and we have subsequently extended this case to the one in which the same region behaves like a visco-elastic Maxwell fluid [3].

We have then studied the case of an elastic material such that no deformation occurs above a certain threshold [4] and we have investigated the case in which the transition from rigid to elastic occurs when the stress becomes greater than the threshold (see [5,6]). The methodology developed in all these papers can be used to formulate a variety of models for continuum with stress threshold.

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