



## On the Iwan models for lap-type bolted joints

Ivan I. Argatov<sup>a</sup>, Eric A. Butcher<sup>b,\*</sup>

<sup>a</sup> Institute of Mathematics and Physics, Aberystwyth University, Ceredigion SY23 3BZ, Wales, UK

<sup>b</sup> Department of Mechanical and Aerospace Engineering, New Mexico State University, Las Cruces, NM 88001, USA

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### ABSTRACT

The paper presents mathematical modeling of the non-linear constitutive relation for bolted joints in the framework of the Kragelsky–Demkin theory of rough contact. It is shown that this approach, which maintains the tribology-related features of bolted joint interfaces, leads to a singular Iwan distribution density. In particular, we show that the Iwan density is expressed in terms of the height distribution density of the surface asperities, whereas its singular exponent is determined by the shape exponent of the surface asperities. Following this, constitutive relations for lap joints and the corresponding backbone (force–deflection) curves are obtained. Finally, Masing's hypothesis is applied and Goodman's relation for energy dissipation is recovered in order to describe the effects of cyclic loading. The two cases of a rough surface in contact with a flat surface and of two contacting rough surfaces are treated separately.

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### 1. Introduction

Energy dissipation due to micro-slip in bolted joints plays an important role in many mechanical processes. In spite of a century-old history of contact mechanics, the subject of bolted joints is an example where our current knowledge of frictional contact interaction is still incomplete. Many papers have focused on constitutive modeling of bolted joints to accommodate non-linearities that are localized at the contact interface (see the review given by Segalman [1]). One approach has been to develop series Iwan models [2] in representing hysteretic contact dissipation [3]. This approach uses singular Iwan distribution densities and employs the measured properties of the bolted joints in order to establish relevant unknown parameters in the model.

It should be noted that the Iwan models are identical to a number of other commonly used hysteresis models for which there has been much analytical work done. Some of this work concerns model identification and inversion, vibratory energy flow and dissipation, and microscopic versus macroscopic interpretations. Specifically, it has been shown that the parallel–series Iwan model is mathematically identical to the Maxwell resistive–capacitor model of hysteresis and the Prandtl–Ishlinskii hysteresis model, and thus is a type of classical Preisach model [4,5]. The application of the Preisach hysteresis formalism for contact mechanics of rough surfaces was recently discussed in [6].

The tribological behavior of the contacting rough surfaces of bolted joints subjected to normal and shear loads depends on the topography of the surfaces. Hence, the topographical characteristics of rough surfaces in contact should influence the

parameters of the joint constitutive models. Approximate constitutive relations for lap joints were recently obtained by Farhang et al. [7] using Greenwood and Williamson's statistical approach [8] for describing contact between two nominally flat rough surfaces. In the present paper, we investigate the generalized non-linear Iwan model associated with the tangential contact of rough surfaces by employing the Kragelsky and Demkin deterministic approach [9] for describing contact between two rough surfaces. We show that the Iwan distribution density can be expressed in terms of the height distribution density of the surface asperities. In the case of the height distribution density satisfying the so-called Demkin power law, we derive the constitutive relation for lap-type bolted joints in explicit form, and we express the singularity exponent of the Iwan distribution density in terms of the shape exponent of the surface asperities. The constitutive relation is obtained on the basis of the Cattaneo–Mindlin theory of tangential contact from which the backbone curves describing the force–deflection relationship are obtained and comparisons are made to a macroscopic asperity model. Finally, Masing's hypothesis is applied to obtain the unloading and reloading curves for cyclic forcing, and a Goodman-type expression for the energy dissipation per loading/unloading cycle is obtained.

### 2. Interpretation of the Iwan distribution density

In this section, the Iwan parallel–series model is briefly described. We express the ultimate friction force and derive an integral representation for this quantity in the framework of the Kragelsky–Demkin theory of rough contact. We consider separately the cases of one rough surface in contact with a flat surface and of two contacting rough surfaces. The formulas obtained for the

\* Corresponding author. Tel.: +1 575 646 6179; fax: +1 575 646 6111.

E-mail addresses: [iva1@aber.ac.uk](mailto:iva1@aber.ac.uk) (I.I. Argatov), [eab@nmsu.edu](mailto:eab@nmsu.edu) (E.A. Butcher).