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Stability of a Rayleigh–Bénard Poiseuille flow for yield stress fluids—Comparison between Bingham and regularized models

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A B S T R A C T

A linear stability analysis of a Rayleigh–Bénard Poiseuille flow is performed for yield stress fluids whether we use the Bingham or regularized models. A fundamental difference between those models is that the effective viscosity is not defined in the plug zone for the Bingham model, while it is defined in the whole domain for the regularized models. For these models, the viscosity depends highly on a parameter ε near the axis and increases drastically in an intermediate region. The convergence of the critical conditions between the simple and the Bingham models is not obtained. However, we show that the Bercovier and Papanastasiou models can tend to the exact Bingham results.

1. Introduction

Different models are used in the literature to describe a yield stress fluid. The simplest one is the Bingham model. It is commonly used to describe the rheological behaviour of a large range of fluids as muds of drilling for instance. It assumes that the material moves as a rigid solid prior to yielding and behaves as a viscous fluid afterwards. By definition, the sol–gel transition for a Bingham fluid is not continuous in terms of material behaviour. The "gel-like" region, called plug or unyielded zone, and the "liquid-like" region are separated by a distinct yield surface. Furthermore, except few simple configurations [1], the determination of the yield surface location is the major difficulty in the numerical resolution of the Bingham fluid flow. To avoid these limitations, several authors use regularized models. These models consider a viscous behaviour in the whole flow domain and replace the unyielded zone by an extremely viscous fluid past a transition in the shear rate. The aim of our study is to compare the different models and show the relevance of using either regularized models or the Bingham model in a particular situation: stability problems. In this paper, we investigate the Rayleigh–Bénard Poiseuille (RBP) flow for yield stress fluids. This configuration has already been studied for the Bingham fluid in [2,3] performing linear stability analyses.

On the other hand, the usage of regularized models in stability analysis has been studied recently by [4,5]. In [4], the authors treat a linearly stable flow: the plane Poiseuille flow of the Bingham fluid. The authors show that the regularized models can exhibit spurious behaviour and can give rise to unstable eigenmodes. The eigenvalues are called spurious in the sense that they depend on the small parameter introduced in the regularized model. These values can be detected by varying the number of nodes in calculations. In [5], the linear stability of the circular Couette flow of viscoplastic fluid leads to critical conditions for both the Bingham and a regularized models. The author indicates that the regularization has practically no influence on critical conditions. Actually, it is not surprising that in the case of plane Poiseuille flow, the regularized model leads to an instability at finite Reynolds number. Indeed, replacing a rigid plug zone by a highly viscous zone (whatever, the large value of the viscosity, i.e. the small is the regularization parameter), leads to a problem fundamentally different from that of a true plane Bingham Poiseuille flow. However, it is not clear in Frigaard and Nouar [4] how the critical conditions depend on the regularization model and on the regularization parameter. The analysis of the viscosity profiles combined with the work of Govindarajan et al. [6], where it is shown that the critical conditions depend mainly on the viscosity stratification in the critical layer (region where the energy of fluctuations is produced by interactions with the mean flow), may lead to the conclusion that the critical conditions are mainly independent on the regularization parameter, when it is sufficiently small.

In the case of the Taylor–Couette problem, where the instability is driven by the centrifugal force, it can be understood here

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