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The Lie-group shooting method for boundary-layer problems with suction/injection/reverse flow conditions for power-law fluids

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ABSTRACT

For power-law fluids we propose a Lie-group shooting method to tackle the boundary-layer problems under a suction/injection as well as a reverse flow boundary conditions. The Crocco transformation is employed to reduce the third-order equation to a second-order ordinary differential equation, and then through a symmetric extension to a boundary value problem with *constant boundary conditions*, which can be solved numerically by the Lie-group shooting method. However, the resulting equation is singular, which might be difficult to solve, and we propose a technique to overcome the initial impulse caused by the singularity using a very small time-step integration at the first few time steps. Because we can express the missing initial condition through a closed-form formula in terms of the weighting factor $r \in (0,1)$, the Lie-group shooting method is very effective for searching the multiple-solutions of a reverse flow boundary condition.

1. Introduction

The boundary-layer theory explains very well the steady-state flow over a flat plate at zero incidence angle known as the Blasius flow. It is long known that the appearance of boundary layers is not restricted to the canonical problem of the motion of a body through a viscous fluid. Several other technologically important sources of boundary-layer behavior are the flows behind expansion and shock waves travelling over smooth surfaces and the flow above a moving conveyor belt.

A main reason for the interest in analysis of boundary-layer flows over solid surfaces is the possibility by applying the theory to the efficient design of supersonic and hypersonic flights. Besides, the mathematical model considered in the present research has importance in studying many problems of engineering, meteorology, and oceanography; see, for example, Schlichting [1], Ozisik [2], Nachman and Callegari [3], Shu and Wilks [4], Hopwell [5], Zheng et al. [6,7], Zheng and Deng [8], Zheng and He [9], Zheng and Zhang [10].

Klemp and Acrivos [11,12] have studied boundary-layer flow caused by a finite flat plate that moves in the direction opposite to the main stream at high Reynolds numbers. They thought that a region of reverse flow remains confined within a boundary-layer and the conventional boundary-layer equations should continue to apply downstream of the point of detachment of the surface streamline. In the upstream portion of the separated region, the boundary-layer equation possesses a similarity solution. Hussaini and Lakin [13] showed that the solutions of such boundary-layer problems exist only up to a certain value of the velocity ratio parameter.

We assume that the moving flat plate is semi-infinite with a porous surface and that the plate is moving at a constant speed U_w in the direction parallel to an oncoming flow with a constant speed U_∞. By the assumption of incompressibility and the conservation of momentum, the laminar flow satisfies

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{1}{\rho}\frac{\partial \tau_{XY}}{\partial Y}. \tag{2}$$

In above, X and Y are the coordinates attached to the plate in the horizontal and perpendicular directions, and U and V are respectively the velocity components of the flow in the X and Y directions. The fluid density ρ is assumed to be a constant.

The shear stress is governed by a power law:

$$\tau_{XY}=K\left|\frac{\partial U}{\partial Y}\right|^{N-1}\frac{\partial U}{\partial Y}, \tag{3}$$

where $K>0$ is a constant and the power $N>0$ reflects the discrepancy to the Newtonian fluids (with $N=1$), where in the case that $N<1$ is the power law of pseudo-plastic fluids and $N>1$ is the dilatant fluids. The corresponding boundary conditions are

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