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The unilateral contact buckling problem of continuous beams in the presence of initial geometric imperfections: An analytical approach based on the theory of elastic stability

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ABSTRACT

An analytical method for the treatment of the elastic buckling problem of continuous beams with intermediate unilateral constraints is presented, which is based on the fundamental theory of elastic stability. The study focuses on the unilateral contact buckling problem of beams in the presence of initial geometric imperfections. The mathematical Euler approach, based on the fundamental solution of the boundary value problem of the buckling of continuous beams, is appropriately modified in order to take into account the unilateral contact conditions. Furthermore, in order the obtained analytical solutions to be applicable for practical design cases, the actual strength of the cross-section of the beam under combined compression and bending is considered. The implementation of the proposed method is demonstrated through a characteristic example.

1. Introduction

In many cases engineers deal with practical problems where, besides classical boundary conditions (bilateral constraints), unilateral support conditions are also possible. It is obvious that classical structural or mechanical problems with a certain level of complexity become even more complex to be solved when unilateral constraints should also be taken into account. Buckling involving unilateral contact is encountered in a variety of different practical applications. The unilateral contact buckling problem of delaminated plates in a composite member, the possible contact buckling phenomena in metal forming processes, and the buckling of the compressive plates of the steel sheeting in a composite slab, are some of the many structural and mechanical problems where the beginning and the evolution of the buckling phenomena are strongly affected by unilateral contact conditions.

Problems of that type are usually handled using computational techniques based on variational formulations of the governing differential equations [1–8]. For the solution of some specific problems in the area of civil engineering, simplified mathematical models have also been developed by Ma et al. [9,10]. Furthermore, some researchers [11–15] have studied the aforementioned problem from a more theoretical point of view, giving analytical solutions for some classes of problems.

In addition, the stability and strength of structures in real life applications is influenced by the existence of initial geometric imperfections which develop due to a variety of reasons, as e.g. manufacturing processes, member handling from the factory to the construction site, etc. For this reason, the design against buckling in all the structural design codes considers initial geometric imperfections having certain shapes and amplitudes.

In the present paper an analytical method is developed which can be applied in common practical problems. The aforementioned methodology is based on the linear elastic stability theory, appropriately extended in order to take into account the unilateral constraints. More specifically, the Euler equilibrium method [16,17] of finding the instability load of continuous beams in the presence of initial geometric imperfections, is applied. The considered continuous beams, besides bilateral constraints, are subjected to unilateral ones.

For the implementation of the proposed method, first arbitrary initial geometric imperfections compatible with the function of the unilateral constraint are introduced in the structure. Then, the governing differential equations describing the bending behavior of the beam are constructed. The boundary conditions of the problem are formulated appropriately, in order to take into account the unilateral constraints. In the sequel, the arising boundary value problem (BVP) is examined under all the possible contact situations. The solution of the BVP gives the deflection curve of the beam as a function of the applied load. The so-called **instability load** is the one that leads the values of the obtained deflection curve of the beam to infinity. On the other hand, it

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