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# Two-layer film flow on a rotating disk: A numerical study

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### 1. Introduction

Development of thin film on horizontal rotating disk is known as spin coating in the literature. This technique is widely used in microelectronics industries to coat the photoresist on silicon wafers for integrated circuits, for magnetic storage disks, for magnetic paint coating on the substrates, etc. Emslie et al. [1], first initiated the study on the development of thin film on a rotating disk. Assuming a balance between the centrifugal and viscous force during the process of rotation they were able to simplify the system of Navier-Stokes equations and finally, able to show that the uniformity of the film is maintained as it thins continuously more and more. Later on Meyerhofer [2], Lai [3], Chen [4], Ma and Hwang [5], and others have studied the different effects like solvent evaporation, surface tension, non-Newtonian effects, disk roughness etc. on film thinning. All these analyses were based on the typical hydrodynamical approximation as employed by Emslie et al. [1]. Full Navier–Stokes equations were considered first time by Higgins [6] to study the flow and film development through matched asymptotic expansion procedure and able to show that the uniformity of the film thickness remains as spinning process continues. Following Higgins's analytical treatment Dandapat and Ray [7,8], Ray and Dandapat [9] have studied the effects of film cooling, thermocapillarity and magnetic field on film development. Dandapat [10] has observed that at the initial stage of rotation, the film thinning rate is more for slow, than the fast rotational speed, but at large time, film thinning rate increases with the faster rotational speed if

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#### ABSTRACT

Development of thin two-layer film over a uniformly rotating disk is studied numerically under the assumption of planar interface and free surface. Similarity transformation is applied to transform the Navier–Stokes equations into a set of coupled non-linear, unsteady partial differential equations. This set of equations are solved numerically by using the finite-difference technique. It is observed that the rate of film thickness varies at different time zone depending on the rate of rotational speed of the disk. A physical explanation is provided to justify this anomalous behaviour. It is observed that, smaller thickness on the top layer enhance the initial rate of film thinning. But the overall effect of density, viscosity and the initial film thickness ratio are found to be insensitive to the final film thickness at large time.

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the spinner starts from rest with impulsive rotation and maintains the speed for the rest of the period. However, faster rate of thinning can be obtained if the spinner starts impulsively and then increases its spinning rate continuously. It is to be noted here that in all the above studies the tacit assumption was that the disk is wet so that the classical no-slip boundary condition can be applied at every point on the disk surface and the film flows under a planar interface for entire period of spinning.

It is to be remembered here that in general, an interface always separates two different physical media. So to understand the spincoating process in details we need to study the film thinning mechanism by considering a two-layer fluid model. This necessity we felt from our past experiences in connection with the other flow problems. There we have seen new phenomena appears when interface between two fluids is considered. This is because in the frame of one fluid model the movement of gas/liquid vapour is treated as passive, whereas in reality the motion of gas/liquid vapour can exert shear stress on the surface of the fluid. This neglected shear stress on the free surface may brings changes in the system. Multilayered approach has been applied for the investigation of several modern engineering processes. For example, the liquid encapsulation technique used in crystal growth [11]. Multilayered approach also used in multilayer coating for the production of photographic films and multilayer fibers for optoelectronics devices [12] and emulsified liquid membrane separation techniques widely used now in various extraction processes and industrial waste-water treatment [13]. Kitamura [14] has advocated the usefulness of multilayered coating in potentially hazardous environment.

To the best of our knowledge, so far, no theoretical study has been reported in the literature of thin film development on a rotating disk considering two or multilayered initial deposition of the liquid.

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