International Journal of Non-Linear Mechanics 46 (2011) 1232–1239

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Load carrying capacity of systems within a global safety perspective. Part I. Robustness of stable equilibria under imperfections

Stefano Lenci [a,*], Giuseppe Rega [b]

[a] Department of Architecture, Buildings and Structures, Polytechnic University of Marche, Ancona, Italy
[b] Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Rome, Italy

ARTICLE INFO

Available online 16 June 2011

Keywords:
Practical stability
Static imperfections
Global safety
Robustness of equilibria
Euler
Koiter
Thompson

ABSTRACT

The problem of the practical stability of structures is addressed in a modern way by considering the effects of both static and dynamic perturbations. The major historical contributions, due to Euler, Koiter and Thompson, are reviewed and illustrated by an archetypal model permitting to highlight the main mechanical and dynamical points. It is found that a global approach is necessary for a reliable safety estimation, especially in the neighborhood of (static) critical loads. Considering that the admissible load threshold has to account for robustness to finite perturbations, the Koiter critical load must be lowered, obtaining the so called Thompson critical load. It is shown how these two thresholds share some properties (e.g., both depend in a sensitive way on imperfections, which must be known for practical calculations), while having a deep different meaning: the former is related to static imperfections, and requires only a local analysis, while the latter is related to dynamical imperfections, and requires a global analysis. It is shown that $P^{crit}_{Euler} \geq P^{crit}_{Koiter} \geq P^{crit}_{Thompson}$, i.e., that the advancement of knowledge leads to a lower estimation of the actual critical load.

1. Introduction

Determining the load carrying capacity of a compressed structural element (columns, frames, shells, etc.) is an old problem, which dates back at least to Euler [1]. It was a long history of successes and defeats of scientists, which, in the authors' opinion, still extends over the present time.

The first fundamental contribution was due to Euler [1], which determined the famous Euler buckling load of a column. The loss of load carrying capacity was identified as the bifurcation point from an equilibrium path in the parameters space—talking, of course, in modern language. Although the concept of stability was formulated in a rigorous way only much later, with the major contribution, among others, of Lyapunov [2], it is felt that the main idea of loss of stability was already lurking in Euler's background. We quote [3] for an interesting historical note on the development of the stability concepts.

The second outstanding contribution was due to Koiter [4], who realized that imperfections are crucial in lowering the critical load. This basic idea and successive developments were so important that investigations in this direction continue up to date [5]. In practice, due to imperfections, the branching point becomes a snap point, which occurs at a lower load threshold. Later on, the bifurcation theory has provided a mathematical background to this engineering intuition, by rigorously showing that transcritical and pitchfork bifurcations (responsible for branching) are *structurally* unstable events, which become saddle-node bifurcations (responsible for snap) after system perturbations (imperfections in mechanical language). Structural stability concepts are part of the more general *catastrophe theory* [6,7]. In this context, though the general theorems are very complex and abstract, the basic idea is simply that of studying perturbations of the system with respect to parameters and not to initial conditions, as in classical local stability.

Although at Koiter's time it was clear that stability is a dynamical concept [8], the major initial contributions were concerned with a "static" stability approach [9,10]. When 'flutter' or 'galloping' came to the attention of researchers, dynamics entered the question of loss of stability (see [11] for a theoretical approach and [12] for a practical approach). In the bifurcation theory language, the Hopf bifurcation was 'discovered' to exist in practice, and this agrees with the fact that it is a *structurally* stable event, which can be actually seen to occur as, e.g., in the dramatic failure of the Tacoma Bridge or in other aero-elastically induced collapses of structures.

This basic, and necessarily over-summarized, evolution of knowledge is clearly understood (see, e.g., [13]), and it is apparent that only *local* bifurcational events were concerned, indeed.

It was Thompson that, around the 90s [14–16], discovered that (local) stability is not enough, and that the relevant results do not

* Corresponding author. Tel.: +39 071 2204552; fax: +39 071 2204576.
E-mail addresses: lenci@univpm.it (S. Lenci), Giuseppe.Rega@uniroma1.it (G. Rega).

0020-7462/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2011.05.020