



Solutions of some simple boundary value problems within the context of a new class of elastic materials

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ABSTRACT

Some simple boundary value problems are studied, for a new class of elastic materials, wherein deformations are expressed as non-linear functions of the stresses. Problems involving 'homogeneous' stress distributions and one-dimensional stress distributions are considered. For such problems, deformations are calculated corresponding to the assumed stress distributions. In some of the situations, it is found that non-unique solutions are possible. Interestingly, non-monotonic response of the deformation is possible corresponding to monotonic increase in loading. For a subclass of models, the strain–stress relationship leads to a pronounced strain-gradient concentration domain in the body in that the strains increase tremendously with the stress for small range of the stress (or put differently, the gradient of the strain with respect to the stress is very large in a narrow domain), and they remain practically constant as the stress increases further. Most importantly, we find that for a large subclass of the models considered, the strain remains bounded as the stresses become arbitrarily large, an impossibility in the case of the classical linearized elastic model. This last result has relevance to important problems in which singularities in stresses develop, such as fracture mechanics and other problems involving the application of concentrated loads.

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1. Introduction

It has been shown recently that if by elastic bodies we mean bodies that are incapable of dissipating energy in any process to which these bodies are subject to, then this class is far larger than Cauchy elastic or Green elastic bodies (see Rajagopal [1,2], Rajagopal and Srinivasa [3,4] for a detailed discussion of the rationale for such models). Elastic bodies need not only be characterized by providing an explicit expression for the stress as a function of the deformation gradient (Cauchy elastic bodies), or by assuming that there exists a stored energy that depends on the deformation gradient (Green elastic bodies). It is possible that elastic bodies could be defined by implicit constitutive relationships between the stress and the deformation gradient, and the stored energy could be a function of both the stresses and the deformation gradient. In fact, the models could be even more general in that one need not even define a deformation gradient in order to define elastic bodies. For instance, Rajagopal and Srinivasa [4] provide rate equations wherein the symmetric part of the velocity gradient and the rate of the Green–Saint Venant strain are related by implicit relations.

Based on the work of Rajagopal [1,2], Bustamante and Rajagopal [5] developed equations governing plane stress and plane strain for constitutive relations wherein the linearized strain bears a non-linear relationship to the stress. Such a model can be rigorously justified within the context of models, where the stretch tensor is a non-linear function of the stress or within the context of theories wherein the stress and the stretch are implicitly related (see Rajagopal [1]). Bustamante and Rajagopal [5] showed that even for constitutive models where the linearized strain depends non-linearly on the stress, one could introduce an Airy stress function which automatically satisfies the equations of equilibrium and the compatibility equation reduces to a non-linear partial differential equation for the Airy stress function. In three-dimensional problems one could introduce the stress potential and the compatibility equations would then reduce to a system of non-linear partial differential equations for the stress potential (see Finzi [6], Truesdell and Toupin [7]).

Even in two-dimensional problems, as the governing equations are very complicated and not amenable to analysis, Bustamante and Rajagopal [5] developed a weak formulation within which numerical calculations could be carried out. Bustamante [8] subsequently extended the analysis in Bustamante and Rajagopal [5], both within the context of developing governing equations as well as numerical analysis. However, in none of the above studies involving this new class of elastic materials were any specific

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