International Journal of Non-Linear Mechanics 46 (2011) 180–185

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Steady state and stability of a beam on a damped tensionless foundation under a moving load

Jen-San Chen*, Yung-Kan Chen

Department of Mechanical Engineering, National Taiwan University, Taipei 10617, Taiwan

ARTICLE INFO

Article history:
Received 15 September 2009
Received in revised form
28 July 2010
Accepted 20 August 2010

Keywords:
Tensionless foundation
Infinite beam
Moving load
Steady state
Stability

ABSTRACT

In the first part of this paper we study the effect of damping on the multiple steady state deformations of an infinite beam resting on a tensionless foundation and under a point load moving with a subcritical speed. Due to the non-linear characteristics of the problem, a guess on the deformed shape has to be made before a numerical search can be initiated. It is found that when the damping is present, all the steady state solutions are asymmetric. As the damping approaches zero, some of the steady state solutions become symmetric, while some others remain asymmetric. In the second part of the paper we propose to test the stability of these steady state deformations by a transient analysis on a long finite beam. Our numerical experiment indicates that among all these multiple steady state solutions only one of them is stable. This stable steady state deformation reduces to a symmetric solution when the damping approaches zero. Furthermore, it is found that this stable solution is also the one among all steady state solutions closest in shape to the linear solution based on a bilateral foundation model.

1. Introduction

In the area of high-speed transportation or rocket-sledge technology, the response of rails and rail beds to high speed moving loads is of interest. This problem can be modeled as a beam on elastic foundation subjected to a concentrated load moving at a constant speed. A recent review of this topic can be found in Wang et al. [1]. The earliest and probably the simplest mathematical model adopted to simulate the rail bed is the Winkler elastic foundation [2]. This conventional elastic foundation can sustain both compression as well as tension when the beam deforms. This model is also called the bilateral foundation. By considering the beam to be infinitely long, Kenney [3] studied the steady state deformation when the point load is moving with constant speed. The effect of viscous damping in the foundation on the response was also discussed.

The bilateral foundation model was probably motivated more by the desire of mathematical simplicity than by physical reality. For instance, a railroad rail actually lifts off the ballast in front of the moving train. Under certain circumstances, this lift-off phenomenon may become an important triggering mechanism for track buckling due to constrained thermal expansion. In order to address this issue a tensionless foundation model was proposed, also called a unilateral foundation. In this model the foundation can sustain compression but not tension. Weitsman [4] conducted an extensive study on the deformation of an infinite beam resting on a tensionless foundation under static loading. He found that the contact length is independent of the magnitude of the applied load. In the case when the point force is moving at a constant speed, Weitsman [5] investigated the conditions under which an infinite beam would separate from the tensionless elastic foundation. Kameswara Rao [6] studied the same problem but focused on the effect of viscous damping in the foundation.

The steady state deformation of an infinite beam on a tensionless elastic foundation under a single moving force was first studied by Choros and Adams [7]. Lin and Adams [8] later extended the solution to the case when multiple moving loads were applied. In these two papers, the foundation was assumed to be undamped. Multiple steady state solutions, both symmetric and asymmetric, were reported.

Recently, Maheshwari et al. [9] proposed that a reinforced granular bed overlying a soft soil strata be used in order to improve the bearing capacity of the rail-foundation system. They combined Winkler spring model and Pasternak shear layer to represent the behavior of the tensionless foundation. Only one steady state solution was obtained. In addition to steady state solutions, the transient behavior of a beam-foundation structure also attracted research interests. Lee [10] studied the transient response a simply supported finite beam with multiple one-sided point supports in between.

As pointed out by Adams and his colleagues, there exist multiple steady state solutions for a beam resting on a tensionless

* Corresponding author. Tel.: +886 233662693; fax: +886 223631755.
E-mail address: jschen@ntu.edu.tw (J.-S. Chen).

0020-7462/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijnonlinmec.2010.08.007