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Double impact periodic orbits for an inverted pendulum ☆

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A B S T R A C T

There exist many types of possible periodic orbits that impact at the walls for the inverted pendulum impacting between two rigid walls. Previous studies only focused on single impact periodic orbits and symmetric periodic orbits that bounce back and forth between the two walls. They respectively correspond to Types I and II orbits in the Chow, Shaw and Rand classification. In this paper we discuss two types of double impact periodic orbits that have not been studied before. The equations need to be solved for double impact orbits are transcendental and it is very hard to see the structure of the solutions. Consequently the analysis of double impact orbits is much more difficult than that of Types I and II orbits. A combination of analytical and numerical methods is employed to investigate the existence, stability and bifurcations of these orbits. Grazing bifurcations, which do not present for Types I and II orbits, are also observed.

1. Introduction

An impact system, where a vibrator collides with one or more rigid walls or with another moving object, is found in many applications, such as impact print hammers [14], rigid blocks [15] and walking machines [16], etc. Being an important class of piecewise smooth (PWS) dynamical systems, impact systems often exhibit very complicated dynamics. Besides the occurrence of all kinds of traditional bifurcations, such as saddle–node bifurcation, Hopf bifurcation as well as homoclinic bifurcation, period doubling bifurcation [5,32–34], impacts also lead to many new types of complicated bifurcation phenomena, such as grazing, sticking and chattering [3,4,6–8,11,27–29,35,36], etc. In general, such kinds of non-standard bifurcations arising from impact systems and other types of PWS systems are difficult to deal with because of the added non-linearities caused by the non-smoothness. In recent decades, the study of those non-standard bifurcations has become very active and some effective general methods have been developed. For instance, normal form calculations for impact oscillators were studied in [1,12] and a general methodology of reducing multidimensional flows to low-dimensional maps for piecewise non-linear oscillators was proposed in [30]. The characteristic of normal form map for soft impact systems was also analyzed in [26]. In fact there is an enormous literature on this subject, in addition to the aforementioned works, see, for example, the monographs [2,20] and the references therein for more on these issues.

In this paper we consider double impact periodic motions (namely, motions which repeat after every second impact) of the inverted pendulum impacting on rigid walls under external periodic excitation as shown in Fig. 1. We can scale the gap size between the two walls to be two and assume that the mathematical model is given by the following piecewise linear (PWL) differential equation:

$$\begin{cases} \ddot{x}+2\alpha\dot{x}-x=\beta\cos\omega t, & \text{as } |x|<1, \\ \dot{x}\mapsto -r\dot{x}, & \text{as } |x|=1, \end{cases} \tag{1.1}$$

where $\alpha>0$ is a linear damping coefficient and $\beta>0$ is the forcing amplitude, $r\in(0,1]$ is the coefficient of restitution representing energy loss during impact.

The PWL system (1.1) was first proposed by Chow and Shaw in [5] and also by Shaw and Rand in [34]. The subharmonic and homoclinic bifurcations and chaos were discussed for (1.1) in [5,34]. The impact inverted pendulum can be used in the modeling of many mechanical devices, such as rings, rigid standing structures, a mooring buoy, etc. [8]. Due to this reason, it has been extensively studied during the last 20 years. The existence and stability of periodic motions were analyzed under impulsive excitation in [23] and under general periodic excitation in [24]. Properties of cross-well chaos were studied in [32] and the problem of chaos control was addressed in [21,22]. The asymptotic analysis of chattering oscillations is presented in [8]. All of these works assume that the motion of the oscillator between the walls is governed by a linear equation. Efforts were also made in [9,10,25] to extend the Melnikov methods for homoclinic and subharmonic bifurcations established for smooth systems to the

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