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Axisymmetric indentation of curved elastic membranes by a convex rigid indenter

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ABSTRACT

Motivated by applications to seed germination, we consider the transverse deflection that results from the axisymmetric indentation of an elastic membrane by a rigid body. The elastic membrane is fixed around its boundary, with or without an initial pre-stretch, and may be initially curved prior to indentation. General indenter shapes are considered, and the load–indentation curves that result for a range of spheroidal tips are obtained for both flat and curved membranes. Wrinkling may occur when the membrane is initially curved, and a relaxed strain-energy function is used to calculate the deformed profile in this case. Applications to experiments designed to measure the mechanical properties of seed endosperms are discussed.

1. Introduction

An elastic shell may be defined as a three-dimensional elastic body with one dimension that is much thinner than the other two [18]. This enables us to consider the deformation of only the mid-plane of the elastic body in this thinner direction. A membrane may then be defined as a shell which has negligible resistance to bending [18,12]. Elastic membranes are commonly found in biological and engineering contexts, where they may span relatively large areas despite having little volume or weight [11].

There are two different classes of non-linear elastic membrane theories and we refer to Haughton [7] for a more complete comparison of the two theories, but give a brief description here. The first of these membrane theories may be called a ‘membrane-like shell’ [18], which takes the shell theory of three-dimensional elasticity and introduces the membrane assumption of no stress in the direction normal to the membrane. In this formulation the thickness is included in the derivation, as the principal stretch in the thickness direction, λ_n, appears in the governing equations, which explicitly allows the membrane to get thinner to conserve mass when the constraint of incompressibility is imposed. This is a common method of treating membranes, and a comprehensive derivation may be found in Libai and Simmonds [18] and Steigmann [38]. A variational treatment may also be considered, for example see Le Dret and Raoult [15].

The second class of membrane theories occurs from considering the mid-plane of the membrane as a two-dimensional sheet of elastic material embedded in three-dimensional space, entirely neglecting thickness effects through the membrane [16]. Nadler and Rubin [22] call this a ‘simple membrane’, and a consequence of this reduction is that the stretch through the membrane is not included in the formulation. The deformation gradient is then two-dimensional, and there are only two strain invariants and two principal stretches, rather than the three that arise in the three-dimensional theory, see Steigmann [36,39] for details.

It has been shown that the simple membrane and membrane-like shell approaches give the same governing equations to leading order [26,7,38], and it is the specification of the constitutive behaviour which varies between the two theories. It is possible to define two-dimensional strain-energy functions which have no counterpart in the three-dimensional theory, as discussed by Haughton [7], as well as to use three-dimensional strain-energy functions in the two-dimensional theory. Further details of such matters may be found in Libai and Simmonds [18] and Steigmann [38].

When considering membrane deformations using either of the above theories, it is important to ensure that the membrane is in a state of tension, with both principal stresses remaining positive throughout the deformation [18]. If compressive (negative) stresses occur then the membrane may wrinkle, a local buckling event

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