



Review

Some new classes of inverse coefficient problems in non-linear mechanics and computational material science[☆]

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ABSTRACT

Three classes of inverse coefficient problems arising in engineering mechanics and computational material science are considered. Mathematical models of all considered problems are proposed within the J_2 -deformation theory of plasticity. The first class is related to the determination of unknown elastoplastic properties of a beam from a limited number of torsional experiments. The inverse problem here consists of identifying the unknown coefficient $g(\xi^2)$ (plasticity function) in the non-linear differential equation of torsional creep $-(g(|\nabla u|^2)u_{x_1})_{x_1} - (g(|\nabla u|^2)u_{x_2})_{x_2} = 2\phi$, $x \in \Omega \subset R^2$, from the torque (or torsional rigidity) $T(\phi)$, given experimentally. The second class of inverse problems is related to the identification of elastoplastic properties of a 3D body from spherical indentation tests. In this case one needs to determine unknown Lamé coefficients in the system of PDEs of non-linear elasticity, from the measured spherical indentation loading curve $\mathcal{P} = \mathcal{P}(\alpha)$, obtained during the quasi-static indentation test. In the third model an inverse problem of identifying the unknown coefficient $g(\xi^2(u))$ in the non-linear bending equation is analyzed. The boundary measured data here is assumed to be the deflections $w_i[\tau_k] := w(\lambda_i; \tau_k)$, measured during the quasi-static bending process, given by the parameter τ_k , $k = \overline{1, K}$, at some points $\lambda_i = (x_1^{(i)}, x_2^{(i)})$, $i = \overline{1, M}$ of a plate. An existence of weak solutions of all direct problems are derived in appropriate Sobolev spaces, by using monotone potential operator theory. Then monotone iteration schemes for all the linearized direct problems are proposed. Strong convergence of solutions of the linearized problems, as well as rates of convergence is proved. Based on obtained continuity property of the direct problem solution with respect to coefficients, and compactness of the set of admissible coefficients, an existence of quasisolutions of all considered inverse problems is proved. Some numerical results, useful from the points of view of engineering mechanics and computational material science, are demonstrated.

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[☆]The results have been announced at the Satellite Conference of International Congress of Mathematicians, 14–17 August 2010, Delhi, India.

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