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A constitutive equation for filled rubber under cyclic loading

Min Liu, Michelle S. Hoo Fatt*

Department of Mechanical Engineering, The University of Akron, Akron, OH 44325-3903, USA

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ABSTRACT

This paper describes experiments and the development of constitutive equations to predict the steady-state response of filled rubber under cyclic loading. An MTS servo-hydraulic machine was used to obtain the dynamic hysteresis curves for a filled rubber compound in uniaxial tension-compression. The material tests were performed with mean strains from -0.1 to 0.1, strain amplitudes ranging from 0.02 to 0.1, and strain rates between 0.01 and 10 s^{-1}. Temporary material set, the Payne effect and rate-dependence were observed from the experimental results. A hyper-viscoelastic constitutive model was developed to characterize the dynamic response of the rubber. A cornerstone of this constitutive modeling was to devise a scheme to evaluate material set and a finite strain, non-linear viscoelastic law from the test data. Predictions of the dynamic hysteresis curves using the proposed constitutive equation were found to be in good agreement with the uniaxial test results.

1. Introduction

Filled rubber compounds are used in tires, engine mounts and air springs because of their ability to dampen out vibrations and impact while undergoing large deformations without break. The loading in these applications is cyclical, and there has been a great deal of research characterizing the response and developing constitutive models for filled rubbers under cyclic loading. Combining constitutive equations with even the most simplistic geometry of and loading on a rubber article can prove to be mathematically intractable. This is due to a combination of inherent non-linearities in the material behavior and the kinematics associated with large deformations and finite strains. Finite element analysis (FEA) is usually employed to analyze the performance of rubber components under cyclic loads. The goal of this study is to develop a three-dimensional constitutive equation to characterize the dynamic response of a rubber compound subjected to cyclic loading. The loading cycle is defined in terms of a given frequency, strain amplitude and mean strain. The resulting constitutive equation could then be implemented in FEA to predict the dynamic response of a rubber component under cyclic loading.

The conventional approach in characterizing the dynamic material properties of rubber is to find the stiffness and damping of the rubber in the frequency domain in terms of a storage and a loss modulus [1,2]. The concept of the storage and loss moduli is derived by modeling the hysteretic response of the rubber subject to sinusoidal deformation with a linear elastic spring and a linear dashpot in series (Maxwell model). Dynamic experiments have shown that these moduli are not only dependent on frequency and temperature, but also on the strain amplitude (Payne Effect [3]).

The storage/loss modulus concept may be extended in the time domain to simulate material response for arbitrary loading history. A problem exists, however, when this concept is used to derive three-dimensional constitutive equations for real rubber behavior. The stress–strain response of rubber is highly non-linear, and a single storage modulus and a single loss modulus do not accurately describe the dynamic response of filled rubber even at strain amplitudes of a few percent. In an attempt to fit experimental results accurately, generalized Maxwell models or Prony series, involving a multitude of elastic springs and dashpots, are used instead [4]. A finite number of Maxwell springs and dashpots are used to describe a discrete relaxation spectrum, which in turn approximates non-linear viscoelastic response. Material constants are found from non-linear regression or curve-fitting with test data. Fitting experimental data this way is very difficult, often resulting in more than 20 material parameters. Haupt and Lion [5] showed that fewer material constants would be needed to describe relaxation functions if viscoelasticity were defined by differential equations of fractional order (fractional derivative). Compact relaxation functions of the Mittag–Leffler type arose in the kernel of the convolution integral for stress using this approach. Non-linear effects were introduced by replacing natural time with a material-dependent time scale. Subsequent studies that follow from this concept can be found in Lion et al. [6] and Lion and Kardelky [7].

A number of constitutive models have also been developed to address the steady-state, rate-dependent behavior of rubber under

* Corresponding author. Tel.: +1 330 972 6308; fax: +1 330 972 6027.
E-mail address: hoofatt@uakron.edu (M.S. Hoo Fatt).

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