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Stochastic jump and bifurcation of a slender cantilever beam carrying a lumped mass under narrow-band principal parametric excitation

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ABSTRACT

A detailed theoretical investigation into the single-mode approximate response of a slender cantilever beam carrying a lumped mass subjected to base narrow-band random excitation is presented for the first time. The method of multiple scales is used and the stochastic jump and bifurcation have been investigated for the principal parametric resonance of the system using the stationary joint probability. Results show that stochastic jump occurs mainly in the region of triple-valued solution. For the frequency-response domain, if the excitation central frequency is a variable and others keep constant, the basic phenomena imply that the higher the frequency, the more probable the jump from the stationary non-trivial branch to the stationary trivial one once the frequency exceeds a certain value. If the bandwidth is a variable and others keep constant, the basic phenomena indicate that the most probable motion is around the non-trivial branch when the bandwidth is smaller, whereas the most probable motion gradually approaches the trivial one when the bandwidth becomes higher. For the force-response domain, there is a region of excitation acceleration within which the joint probability density has two peaks: an outer flabellate peak and a central volcano peak. Results show that the outer flabellate peak decreases while the central volcano peak increases as the value of the excitation acceleration decreases.

1. Introduction

Slender beams carrying lumped masses subject to base excitation can find application in the modeling of many engineering fields such as tall buildings, robotic manipulators, components of high-speed machinery, accelerating missiles, appendages of aircrafts, spacecrafts or even vehicles, etc. Hence, attention has been paid to the non-linear dynamics of such beams subject to base excitation in a number of technical papers over the past few decades due to both theoretic and practical demands.

Though there are a number of studies available on the beams aforementioned, some of them conventionally focused on the determination of dynamic modeling, natural frequencies and linear mode shapes [1–5]. Lee [6] investigated the stability of a cantilever beam with tip mass subjected to axial harmonic excitation. Sato et al. [7] considered the effect of weight, rotary inertia, location and vibration amplitude of concentrated mass on the natural frequencies of simply supported horizontal beam carrying a concentrated mass. They also dealt with the single-mode principal parametric response using the harmonic balance method. Saito and Koizumi [8] studied the non-linear parametric vibration using the same model and the method as those in Ref. [7] under the influence of gravity. Hamdan and Dado [9] analyzed the large amplitude free vibration of a slender, inextensible cantilever beam carrying a lumped mass with rotary inertia at an intermediate position along its span. Al-Qaisia et al. [10] considered the steady state response of a cantilever beam immersed in a fluid and carrying an intermediate mass subjected parametric excitation. Al-Qaisia and Hamdan [11] extended the work of Ref. [10] and focused on the analysis of the bifurcations and chaos of the same system. Zavodney and Nayfeh [12] derived the non-linear partial differential equation for a slender cantilever beam carrying a lumped mass at an arbitrary position. They compared the resulting planner version of the equations with those in Ref. [13] and found no difference between these two modeling methods. Then, they found theoretically as well as experimentally the non-linear response of the beam using single-mode approximation and method of multiple scales. Though the metallic beams failed prematurely in experiment, the experimental frequency-response curves for three levels of excitation amplitude of a composite beam and for two levels of excitation amplitude of a more flexible and carefully heat treated, tempered and polished steel beam were finally obtained. However, results in figures 10 and 16 in their research show that the frequency-response curve is a definite overhang, i.e., jumps from the higher branch (non-trivial) to the lower one (trivial) or vice versa, and further explanation on the jump phenomena was also not dealt with. Following the equation of motion of the beam described in Ref. [12],

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