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Spherical inflation of a class of compressible elastic bodies

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ABSTRACT

Using a special model that belongs to a new class of elastic bodies wherein the Cauchy–Green stretch is given in terms of the Cauchy stress and its invariants, within the context of the spherical inflation of a spherical annulus, we show that interesting phenomena like the development of "stress boundary layers" manifest themselves. We consider two cases of boundary value problems, one in which there is a cavity in a sphere and the other in which there is a rigid spherical inclusion in a sphere. We show that in the case of a rigid inclusion, it is possible for a pronounced "stress boundary" layer to develop, in that the values of the stresses within this boundary layer that is adjacent to a spherical inclusion are much larger than external to it. We also show that in the case of both the cavity and a rigid inclusion, the stress concentration is an order of magnitude higher than the increase in the deformation gradient, that is, the stress and the stretch do not scale in a similar manner. While the stress adjacent to a rigid inclusion can be 2500 times the applied radial stress, the maximum stretch, which occurs at the rigid inclusion is about 10. While the variation in the stresses are linear in thin walled annular regions, we find that in thick walled annular regions, the variation of the stresses is non-linear.

1. Introduction

A body is said to be Cauchy elastic if the Cauchy stress is determined by the deformation gradient that the body suffers and a body is said to be Green elastic if there exists a stored energy associated with the body that depends only on the deformation gradient which acts as a potential from which the stress can be determined. In both these theories, the stress can be explicitly expressed as a function of the deformation gradient. From a thermodynamic viewpoint, bodies are said to be elastic if they are incapable of dissipation, but from the traditional perspective, there is also a tacit assumption that in all such materials the stress can be expressed as a function of the deformation gradient. Recently, it has been shown that if the only requirement for an elastic body is that the body is incapable of dissipation, then the class of elastic bodies is much larger than either, Cauchy or Green elastic bodies (see [1–6]). It is possible that the stress and the deformation gradient are given by an implicit relationship, with neither the stress being a function of the deformation gradient nor the deformation gradient being a function of the stress. Green [7] in his seminal paper observed that if a Cauchy elastic body that is not Green elastic, then such a body can be used to create a perpetual motion machine. Later Rivilin [8] picked up on the same idea but despite these cautionary remarks, the issue has not received the attention it deserves. Green and Naghdi [9] appealed to the Clausius–Duhem inequality to conclude Cauchy elastic bodies ought to be Green elastic, while Casey [10] showed that the second law of thermodynamics implies that the stress can be derived from a potential.

The classical Cauchy elastic model and Green elastic models are special sub-classes of the more general class of implicit constitutive relations. Another special sub-class are models wherein the deformation gradient is a function of the stress. Standard arguments in continuum mechanics will lead to the Cauchy–Green stretch tensor depending on the stress. It is to a special deformation of such a class of models that this paper is devoted to. Simple problems such as uniaxial state of stress, state of simple shear, etc., have been studied within the context of such models (see [1,11]). Also, Rajagopal and Saccomandi [12] studied a model in which the material moduli depend on the pressure, which leads to an implicit model. They were able to explain certain experiments wherein the material moduli of the materials depend on the pressure by appealing to an implicit model which implies limiting chain extensibility. Interestingly, it is possible within the class of such models to have the stretch tensor have a limiting value while the stress tends to infinity.

Studying boundary value problems within the context of models wherein the Cauchy–Green stretch is a function of the stress is markedly different from the classical approach wherein an explicit representation is given for the stress in terms of the

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